A NNLO QCD study of diphoton production at the LHC

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Indian Institute of Technology Hyderabad – March 1st 2018

Based on:

S. Catani, L. Cieri, D. de Florian, G.F. & M. Grazzini,

arXiv:1110.2375 & 1802.02095

Motivations

Photon pairs or *diphotons* ($\gamma\gamma$) production at high invariant mass ($M_{\gamma\gamma}$) very relevant at hadron colliders.

- Experimentally very clean finale states. Photon energies/momenta measured with high precision.
- Photons not interact strongly: ideal probes for study Standard Model (SM) interactions.
- At the LHC diphotons final states played a crucial role in the Higgs boson discovery ($H \to \gamma\gamma$).
- Diphotons measurements important in searches for physics beyond the SM.

The above reasons and **precise experimental LHC data** demands for **accurate theoretical predictions** $\Rightarrow$ **computation of higher-order QCD corrections**.

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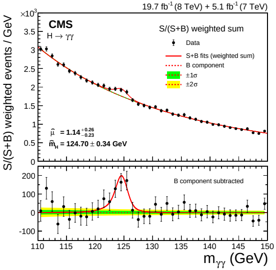
Motivations: Higgs boson studies

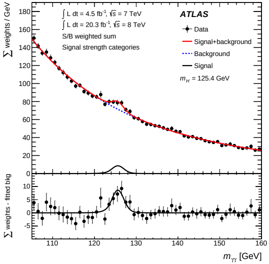
Precise measurements of the Higgs boson properties is a central issue in collider physics.

For $m_H \lesssim 140$ GeV the preferred search mode at the LHC is:

$$\mathbf{gg} \to \mathbf{H} + \mathbf{X} \to \gamma\gamma + \mathbf{X}$$

$\mathbf{pp} \to \gamma\gamma + \mathbf{X}$ is the main irreducible background.

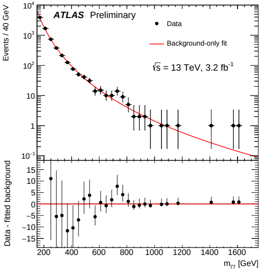




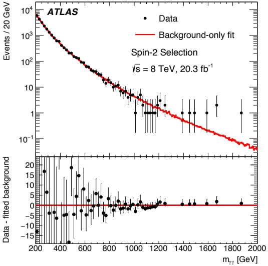
Motivations: beyond the SM searches

Diphoton measurements important in many new-physics scenarios (e.g. searches for extra dimensions or supersymmetry).

In 2015 observation by ATLAS and CMS (2015) of an excess of events (*bump*) in the $M_{\gamma\gamma} \simeq 750$ GeV region.
Excess disappeared (2016) with higher statistics data samples.
It raised a great deal of attention (*bubble*) from theorist:
$\mathcal{O}(10^2)$ of papers, $\mathcal{O}(10^4)$ citations in six months...



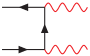
Winter 2015



Summer 2016

Photon production

- PRIMARY or PROMPT photons
  - DIRECT photons
    Directly produced in the hard scattering



$pQCD(+PDFs)$

  - FRAGMENTATION photons Collinear fragmentation of partons into photons

$pQCD(+PDFs) + PFFs$

Only the sum of Direct + Fragmentation component has a physical meaning, given a proper factorization scheme (e.g. $\overline{MS}$)

$$\sigma = \sigma_\gamma(M_F^2) + \sum_p \sigma_p(M_F^2) \otimes D_{p/\gamma}(M_F^2)$$

$D_{a/\gamma}(M_F^2)$ Fragmentation function of a parton p in a γ:
non-perturbative initial condition + Altarelli-Parisi perturbative evolution.

- SECONDARY (NON PROMPT) photons From decays of hadrons (π^0, η) at large p_T or faked by jets.
  Several order of magnitude larger than PROMPT photons
  $\Rightarrow$ Photon isolation is necessary to enhance signal-background ratio

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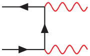

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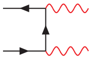

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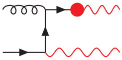

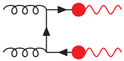

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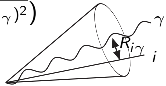
Photon Isolation

- Standard Cone: in a cone of radius R around $\mathbf{p}_\gamma$ the hadronic transverse energy $E_T^{had}(R) \equiv \sum_i E_{T_i}^{had} \Theta(R - R_{i\gamma})$ (with $R_{i\gamma} = \sqrt{(y_i - y_\gamma)^2 + (\phi - \phi_\gamma)^2}$)



$$E_T^{had}(R) \leq E_{T_{max}}$$

  ☹ Not possible to set $E_{T_{max}} = 0$ (to kill fragmentation component): it is not Infrared Safe (soft gluons cannot be emitted inside the cone).

- Smooth Cone[Frixione('98)]: for ALL cones with radius $r < R$ around $\mathbf{p}_\gamma$

$$E_T^{had}(r) \leq E_{T_{max}} \, \chi(r;R) \xrightarrow{r \to 0} 0$$

  ☺ It is Infrared Safe (soft gluons can always be emitted inside the cone).
  ☺ Completely kill (poorly known) Fragmentation component.
  ☺ Direct component well defined (no parton-photon collinear divergences).
  ☹ Not easy to implement (a discrete version) in experimental analyses.

If isolation tight enough NLO QCD predictions with standard and smooth cone are similar (differences smaller than perturbative uncertainties).

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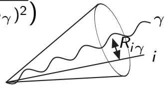

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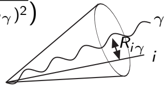

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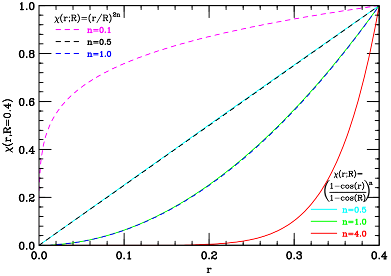
Photon Isolation

Isolation functions:

$$\chi(r;R) = \left(\frac{1-\cos r}{1-\cos R}\right)^n$$

$$\chi(r;R) = \left(\frac{r}{R}\right)^{2n}$$



Shapes $\chi(r;R)$ for various values of power n and $R = 0.4$.

Physical constraints:

- $d\sigma_{\rm smooth}(R; E_{T_{max}}) < d\sigma_{\rm standard}(R; E_{T_{max}})$,
- $d\sigma_{\rm is}(R; E_{T_{max}})$ monotonically decreases as $E_{T_{max}}$ decreases (R fixed) ,
- $d\sigma_{\rm is}(R; E_{T_{max}})$ monotonically increases as R decreases ($E_{T_{max}}$ fixed) ,
- $d\sigma_{\rm smooth}(R; E_{T_{max}}; n)$ monotonically decreases as n increases (R and $E_{T_{max}}$ fixed) ,

Diphoton production

- DIPHOX: NLO QCD for Direct and Fragmentation contributions + part of NNLO ($gg \to \gamma\gamma$ Box) [Binoth,Guillet,Pilon,Werlen('99)].
- gamma2MC: NLO QCD for Direct contribution + part of NNLO ($gg \to \gamma\gamma$ Box) + part of N^{3}LO (corrections to $gg \to \gamma\gamma$ Box) [Bern,Dixon,Schmidt('02)].
- MCFM: LO QCD for Fragmentation contribution + NLO QCD for Direct contribution + part of NNLO ($gg \to \gamma\gamma$ Box) + part of N^{3}LO (corrections to $gg \to \gamma\gamma$ Box) [Campbell,Ellis,Williams('11)].
- NNLL q_T resummation implemented in ResBos [Balazs,Berger,Nadolsky,Yuan ('07)] and in 2γRes [Cieri,Coradeschi,de Florian('15)].
- Lowest order EW corrections computed by [Bierweiler et al.('13)] and [Chiesa et al.('17)]

Diphoton production at NNLO QCD

A complete NNLO in QCD ($\mathcal{O}(\alpha_S^2)$) calculation of both direct and fragmentation components not available.

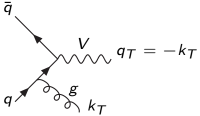
Fragmentation component absent by considering smooth cone isolation. Only direct component needed.

- 2γNNLO: first full NNLO QCD calculation for Direct contribution [Catani,Cieri,de Florian,G.F.,Grazzini ('11)] performed within q_T subtraction formalism (independently implemented in the MATRIX generator [Grazzini,Kallweit,Wiesamann('17)]).
- Independent NNLO QCD calculation for direct contribution within N-jettiness subtraction performed by [Campbell,Ellis,Li,Williams('16)].

The q_T-subtraction formalism at NNLO

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M, q_T) + X$$

V is one or more colourless particles (vector bosons, leptons, photons, Higgs bosons,...) [Catani,Grazzini('07)].



- Key point I: at LO the q_T of the V is exactly zero.

$$d\sigma^{V}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{V+\text{jets}}_{(N)LO} \;,$$

  for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- The only remaining NNLO singularities are associated with the $q_T \to 0$ limit.
- Key point II: treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani,de Florian,Grazzini('00)].

$$d\sigma^{V}_{N^nLO} \xrightarrow{q_T \to 0} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2 = d\sigma^{V}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} \, d^2 q_T$$

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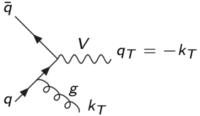

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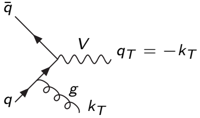

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$$d\sigma^{V}_{(N)NLO} = \mathcal{H}^{V}_{(N)NLO} \otimes d\sigma^{V}_{LO} + \left[d\sigma^{V+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right] \quad ,$$

$$\textit{where} \quad \mathcal{H}^{V}_{NNLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \overset{q_T \to 0}{\longrightarrow} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2$ where $\Sigma(q_T/M)$ is universal.
- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{V+\text{jets}}$: *double real* and *real-virtual* NNLO contributions, while *two-loops virtual* corrections are contained in $\mathcal{H}^{V}_{NNLO}$.
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Diphoton production in NNLO QCD: 2γNNLO

- A NLO calculation requires:
  - $d\sigma_{LO}^{V+\text{jets}}$ (and $d\sigma_{LO}^{V}$).
  - $\mathcal{H}^{V(1)}$ [de Florian, Grazzini('01)]: contains the finite-part of the one-loop amplitude $c\bar{c} \to V$.
  - $d\sigma_{LO}^{CT}$: depends by the (universal) q_T-resummation coeff. A_1 and B_1.
- A NNLO calculation requires also:
  - $d\sigma_{NLO}^{V+\text{jets}}$.
  - $\mathcal{H}^{V(2)}$: contains the finite-part of the two-loops amplitude $c\bar{c} \to V$.
  - $d\sigma_{NLO}^{CT}$: depends by the (universal) q_T-resummation coeff. A_2 and B_2.
- Diphoton production at NNLO within q_T-subtraction:
  - $d\sigma_{NLO}^{\gamma\gamma+\text{jets}}$: [Del Duca,Maltoni,Nagy,Trocsany('03),NLOJet++].
  - $\mathcal{H}^{\gamma\gamma(2)}$ [Catani,Cieri,de Florian,G.F.,Grazzini], and two-loops amplitude for $c\bar{c} \to \gamma\gamma$ [Anastasiou,Glover,Tejeda-Yeomans('02)].

Fully-exclusive NNLO calculation, implemented in the parton-level Monte Carlo code: 2γNNLO [Catani,Cieri,de Florian,G.F.,Grazzini('11)].

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- Diphoton production at NNLO within q_T-subtraction:
  - $d\sigma^{\gamma\gamma+\text{jets}}_{NLO}$: [Del Duca,Maltoni,Nagy,Trocsany('03),NLOJet++].
  - $\mathcal{H}^{\gamma\gamma(2)}$ [Catani,Cieri,de Florian,G.F.,Grazzini], and two-loops amplitude for $c\bar{c} \to \gamma\gamma$ [Anastasiou,Glover,Tejeda-Yeomans('02)].

Fully-exclusive NNLO calculation, implemented in the parton-level Monte Carlo code: 2γNNLO [Catani,Cieri,de Florian,G.F.,Grazzini('11)].

Diphoton production in NNLO QCD: 2γNNLO

- A NLO calculation requires:
  - $d\sigma_{LO}^{V+\text{jets}}$ (and $d\sigma_{LO}^{V}$).
  - $\mathcal{H}^{V(1)}$ [de Florian, Grazzini('01)]: contains the finite-part of the one-loop amplitude $c\bar{c} \to V$.
  - $d\sigma_{LO}^{CT}$: depends by the (universal) q_T-resummation coeff. A_1 and B_1.
- A NNLO calculation requires also:
  - $d\sigma_{NLO}^{V+\text{jets}}$.
  - $\mathcal{H}^{V(2)}$: contains the finite-part of the two-loops amplitude $c\bar{c} \to V$.
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Diphoton production in NNLO QCD: 2γNNLO

The q_T-subtraction formalism cannot deal with IR divergences in the final state
$\Rightarrow$ we rely on Frixione smooth cone isolation (no Fragmentation component) and we calculated the fully exclusive NNLO corrections for Direct component.

Higher order corrections known to be very large:

Box contribution (part of NNLO) large as Born [Dicus, Willenbrock('88)].
Important to have a full control of all the NNLO ($\mathcal{O}(\alpha_S^2)$) contributions:

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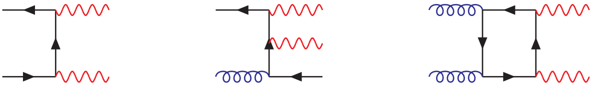

$\hat{\sigma} \sim \alpha^2;\ \mathcal{L} \sim q\bar{q}$ $\quad$ $\hat{\sigma} \sim \alpha^2\alpha_S;\ \mathcal{L} \sim qg$ $\quad$ $\hat{\sigma} \sim \alpha^2\alpha_S^2;\ \mathcal{L} \sim gg$

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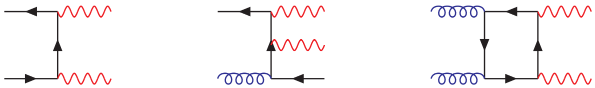

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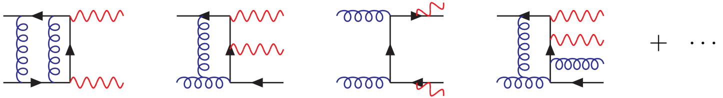

Fiducial cross sections at LO and NLO

Kinematical cuts (ATLAS):
$p_{T\gamma}^{\rm hard} \geq 25$ GeV, $p_{T\gamma}^{\rm soft} \geq 22$ GeV, $|y_\gamma| < 2.37$, $R_{\gamma\gamma}^{\rm min} = 0.4$.

Set up: $\alpha = 1/137$, MMHT 2014 PDFs, BFG-II photon Frag. Funct.,

Scale choice: $\mu_F = \mu_R = \mu_{frag} = \mu_0 \equiv M_{\gamma\gamma}$

Scale variations: $\{\mu_R = \mu_0/2,\ \mu_F = \mu_{frag} = 2\mu_0\}$ and $\{\mu_R = 2\mu_0$, $\mu_F = \mu_{frag} = \mu_0/2\}$ (equivalent to independent variation by a factor 2).
Isolation: $R = 0.4$, $n = 1$.

	$E_{T_{max}} = 2$ GeV		$E_{T_{max}} = 10$ GeV	
	$\sigma^{\rm LO}$ (pb)	$\sigma^{\rm NLO}$ (pb)	$\sigma^{\rm LO}$ (pb)	$\sigma^{\rm NLO}$ (pb)
Standard	$12.15\ ^{+14.5\,\%}_{-14.3\,\%}$	$31.1\ ^{+12.8\,\%}_{-12.3\,\%}$	$19.51\ ^{+25.0\,\%}_{-20.8\,\%}$	$33.3\ ^{+12.3\,\%}_{-11.3\,\%}$
[*direct*]	$10.56\ ^{+10.7\,\%}_{-12.0\,\%}$	$27.30\ ^{+7.8\,\%}_{-9.2\,\%}$	$10.56\ ^{+10.7\,\%}_{-12.0\,\%}$	$18.45\ ^{-10.3\,\%}_{+3.8\,\%}$
Smooth	$10.56\ ^{+10.7\,\%}_{-12.0\,\%}$	$31.92\ ^{+12.6\,\%}_{-12.1\,\%}$	$10.56\ ^{+10.7\,\%}_{-12.0\,\%}$	$33.91\ ^{+13.0\,\%}_{-12.6\,\%}$

Fiducial cross sections at LO and NLO

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$p_{T\gamma}^{\mathrm{hard}} \geq 25$ GeV, $p_{T\gamma}^{\mathrm{soft}} \geq 22$ GeV, $|y_\gamma| < 2.37$, $R_{\gamma\gamma}^{\mathrm{min}} = 0.4$.

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Isolation: $R = 0.4$, $n = 1$.

	$E_{T_{max}} = 2$ GeV		$E_{T_{max}} = 10$ GeV	
	σ^{LO} (pb)	σ^{NLO} (pb)	σ^{LO} (pb)	σ^{NLO} (pb)
Standard	$12.15\ ^{+14.5\,\%}_{-14.3\,\%}$	$31.1\ ^{+12.8\,\%}_{-12.3\,\%}$	$19.51\ ^{+25.0\,\%}_{-20.8\,\%}$	$33.3\ ^{+12.3\,\%}_{-11.3\,\%}$
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Isolation: $R = 0.4$, $n = 1$.

	$E_{T_{max}} = 2$ GeV		$E_{T_{max}} = 10$ GeV	
	σ^{LO} (pb)	σ^{NLO} (pb)	σ^{LO} (pb)	σ^{NLO} (pb)
Standard	$12.15\ ^{+14.5\%}_{-14.3\%}$	$31.1\ ^{+12.8\%}_{-12.3\%}$	$19.51\ ^{+25.0\%}_{-20.8\%}$	$33.3\ ^{+12.3\%}_{-11.3\%}$
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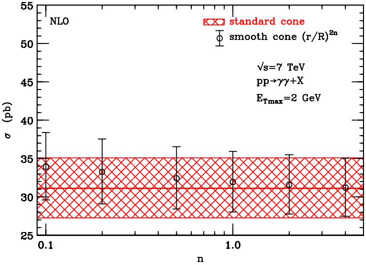
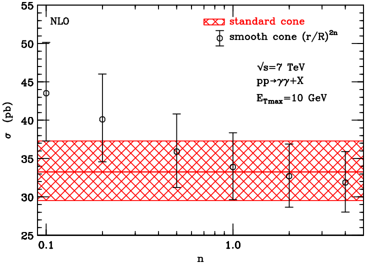


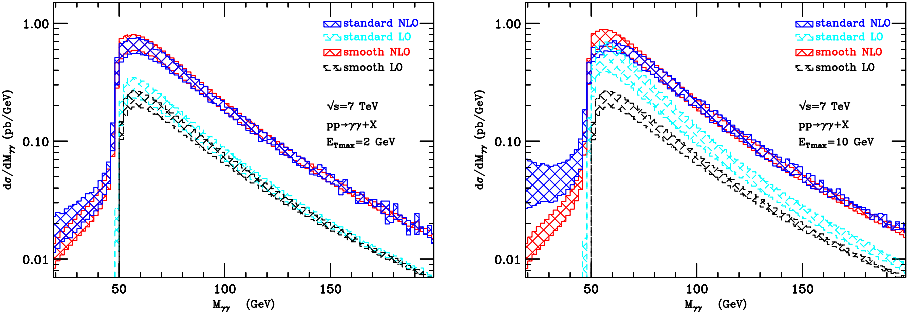


NLO total cross section (with scale variation), for the standard (red line and band) and smooth (black error bars) isolation with $E_{T_{max}} = 2$ GeV (left panel) and 10 GeV (right panel). For smooth cone isolation, various powers of n ($n = 0.1, 0.2, 0.5, 1, 2, 4$) in the isolation function $\chi(r; R) = (r/R)^{2n}$ are considered.

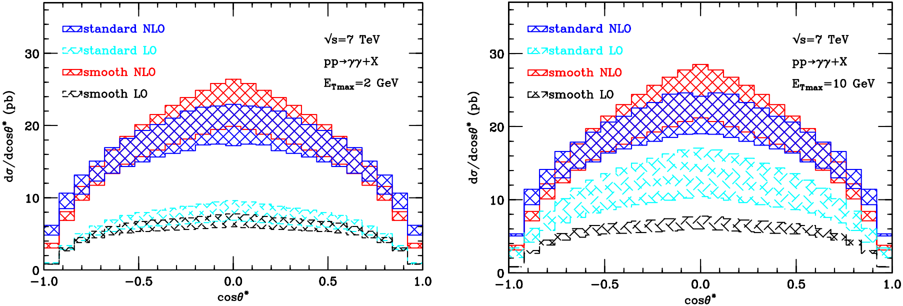
Analytic behaviour of NLO correction for smooth cone isolation in the $n \gg 1$ (soft) and $n \ll 1$ (collinear limit).

$$\delta_{\rm smooth}^{NLO,\rm soft} \propto -\alpha_S R^2 \left(\ln\left(\frac{Q}{E_{T_{max}}}\right) + n \right) , \quad (n \gg 1) ,$$

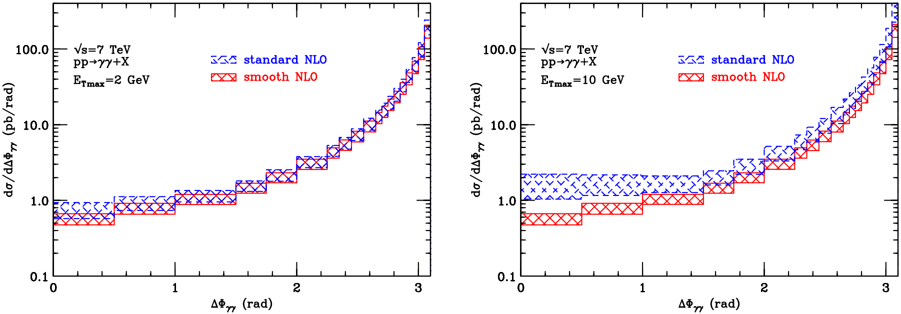
$$\delta_{\rm smooth}^{NLO,\rm coll} \propto +\frac{\alpha_S}{n} \frac{E_{T_{max}}}{Q} , \quad (n \ll 1) .$$



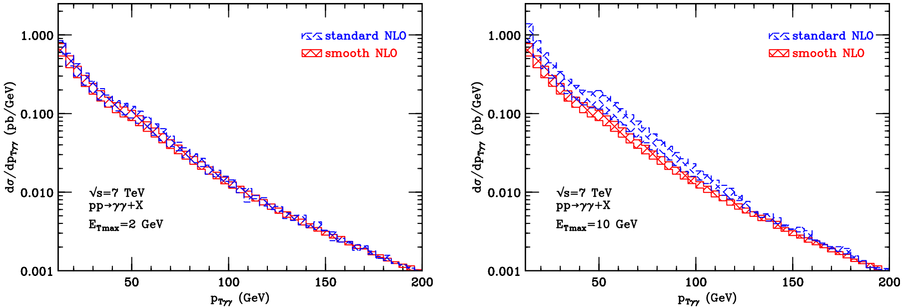
The $M_{\gamma\gamma}$ differential cross section for $E_{T_{max}} = 2$ GeV (left) and $E_{T_{max}} = 10$ GeV (right) at LO and NLO including scale variation bands.



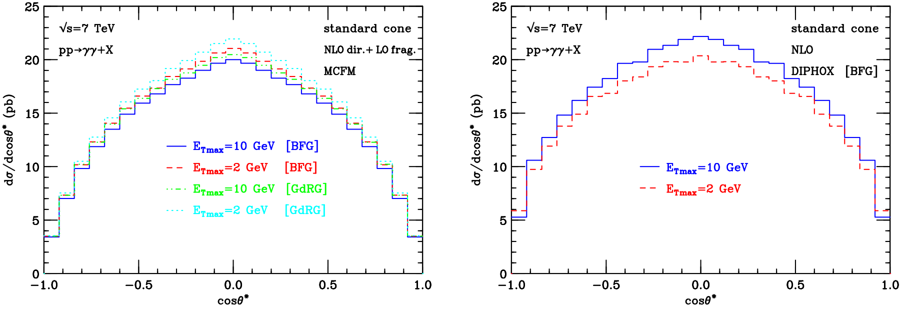
The $\cos\theta^*$ differential cross section for $E_{T_{max}} = 2$ GeV (left) and $E_{T_{max}} = 10$ GeV (right) at LO and NLO including scale variation bands. Where θ^* is the photon polar angle in the Collins-Soper rest frame of the diphoton system.



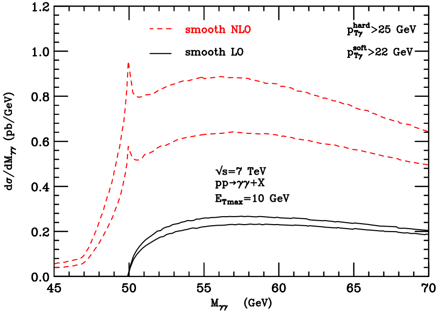
The NLO results (scale variation bands) for the $\Delta\Phi_{\gamma\gamma}$ differential cross section that are obtained by using the smooth (red solid band) and standard (blue dashed band) cone isolation criteria with $E_{T_{max}} = 2$ GeV (left) and $E_{T_{max}} = 10$ GeV (right).



The NLO results (scale variation bands) for the $p_{T\gamma\gamma}$ differential cross section that are obtained by using the smooth (red solid band) and standard (blue dashed band) cone isolation criteria with $E_{T_{max}} = 2$ GeV (left) and $E_{T_{max}} = 10$ GeV (right).



The cos θ^* differential cross section for standard cone isolation with two different values of $E_{T_{max}}$ (2 GeV and 10 GeV). The QCD results are obtained at the central value of the scales ($\mu_F = \mu_R = \mu_{frag} = \mu_0 \equiv M_{\gamma\gamma}$). The results with NLO direct + LO fragmentation components (left) use BFG and GdRG_LO fragmentation functions. The NLO results (right) use BFG fragmentation functions.



The differential cross section $d\sigma/dM_{\gamma\gamma}$ for smooth isolation with $E_{T_{max}} = 10$ GeV. The LO (black solid) and NLO (red dashed) numerical results use $M_{\gamma\gamma}$ bins with constant size of 0.1 GeV. At both perturbative orders, the maximum and minimum values of $d\sigma/dM_{\gamma\gamma}$ correspond to the scale choices $\{\mu_R = M_{\gamma\gamma}/2, \mu_F = 2M_{\gamma\gamma}\}$ and $\{\mu_R = 2M_{\gamma\gamma}, \mu_F = M_{\gamma\gamma}/2\}$, respectively.

Fiducial cross sections at NNLO

Kinematical cuts (ATLAS): $p_{T\gamma}^{\mathrm{hard}} \geq 25$ GeV, $p_{T\gamma}^{\mathrm{soft}} \geq 22$ GeV, $|y_\gamma| < 1.37$ and $1.52 < |y_\gamma| \leq 2.37$, $R_{\gamma\gamma}^{\mathrm{min}} = 0.4$.

Set up: $\alpha = 1/137$, MMHT 2014 PDFs,

Scale choice: $\mu_F = \mu_R = \mu_{frag} = \mu_0 \equiv \sqrt{{M_{\gamma\gamma}}^2 + p_{T\gamma\gamma}^2} = M_{T\gamma\gamma}$

Scale variations: $\{\mu_R = \mu_0/2,\ \mu_F = \mu_{frag} = 2\mu_0\}$ and $\{\mu_R = 2\mu_0$, $\mu_F = \mu_{frag} = \mu_0/2\}$ (equivalent to independent variation by a factor 2).
Isolation ATLAS: cone isolation $R = 0.4$ and $E_{T_{max}} = 4$ GeV.
Isolation NNLO: smooth cone isolation $R = 0.4$ and $E_{T_{max}} = 4$ GeV.

	σ^{LO} (pb)	σ^{NLO} (pb)	σ^{NNLO} (pb)
n ind.	$9.293\ ^{+10.9\,\%}_{-11.9\,\%}$		
$n = 0.5$		$29.40\ ^{+12.8\,\%}_{-12.4\,\%}$	$40.98(68)\ ^{+8.3\,\%}_{-8.7\,\%}$
$n = 1$		$28.55\ ^{+12.5\,\%}_{-12.2\,\%}$	$39.50(50)\ ^{+7.9\,\%}_{-8.4\,\%}$
$n = 2$		$27.98\ ^{+12.3\,\%}_{-11.9\,\%}$	$37.53(52)\ ^{+7.0\,\%}_{-7.8\,\%}$

Results for LO, NLO and NNLO total cross sections.

Fiducial cross sections at NNLO

Kinematical cuts (ATLAS): $p_{T\gamma}^{\rm hard} \geq 25$ GeV, $p_{T\gamma}^{\rm soft} \geq 22$ GeV, $|y_\gamma| < 1.37$ and $1.52 < |y_\gamma| \leq 2.37$, $R_{\gamma\gamma}^{\rm min} = 0.4$.

Set up: $\alpha = 1/137$, MMHT 2014 PDFs,

Scale choice: $\mu_F = \mu_R = \mu_{frag} = \mu_0 \equiv \sqrt{{M_{\gamma\gamma}}^2 + p_{T\gamma\gamma}^2} = M_{T\gamma\gamma}$

Scale variations: $\{\mu_R = \mu_0/2,\ \mu_F = \mu_{frag} = 2\mu_0\}$ and $\{\mu_R = 2\mu_0\ ,$ $\mu_F = \mu_{frag} = \mu_0/2\}$ (equivalent to independent variation by a factor 2).
Isolation ATLAS: cone isolation $R = 0.4$ and $E_{T_{max}} = 4$ GeV.
Isolation NNLO: smooth cone isolation $R = 0.4$ and $E_{T_{max}} = 4$ GeV.

	$\sigma^{\rm LO}$ (pb)	$\sigma^{\rm NLO}$ (pb)	$\sigma^{\rm NNLO}$ (pb)
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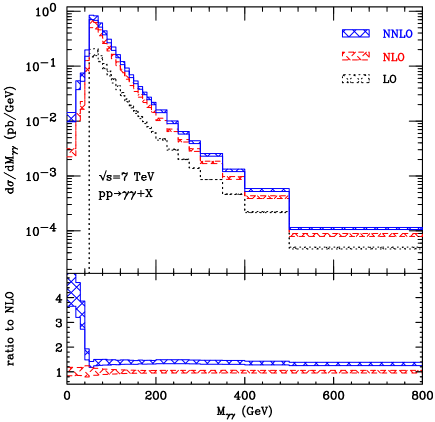
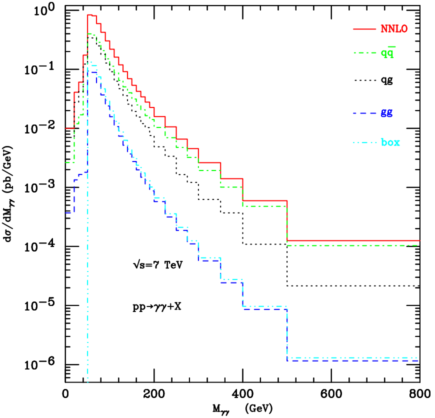
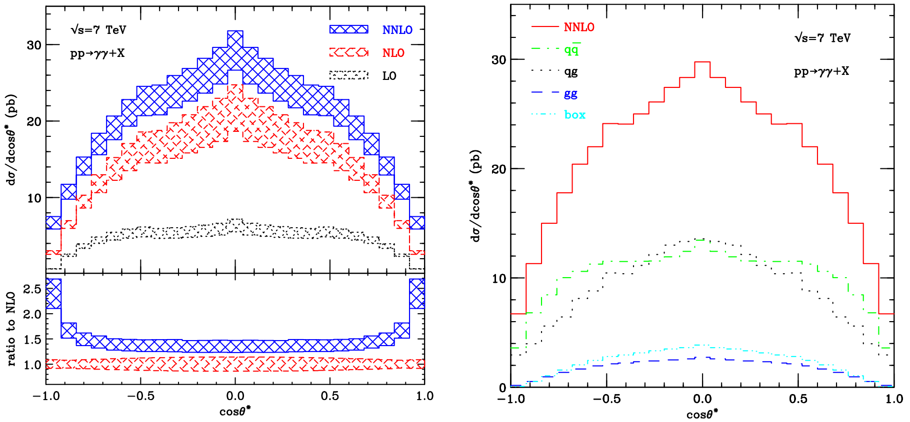
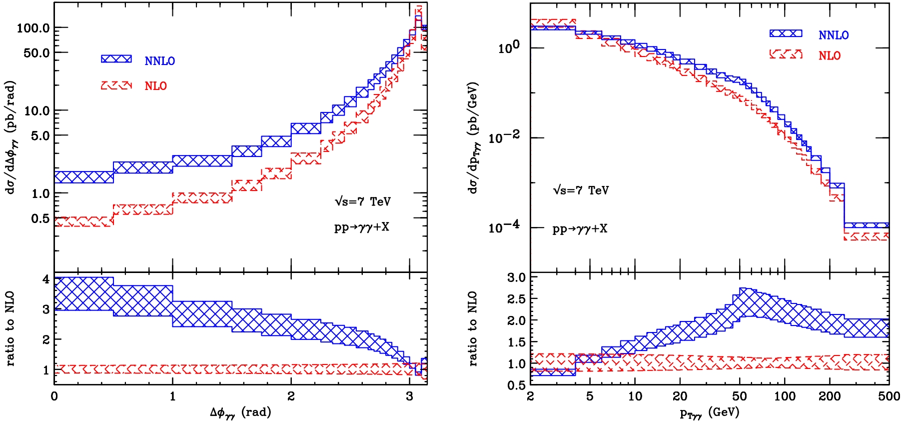




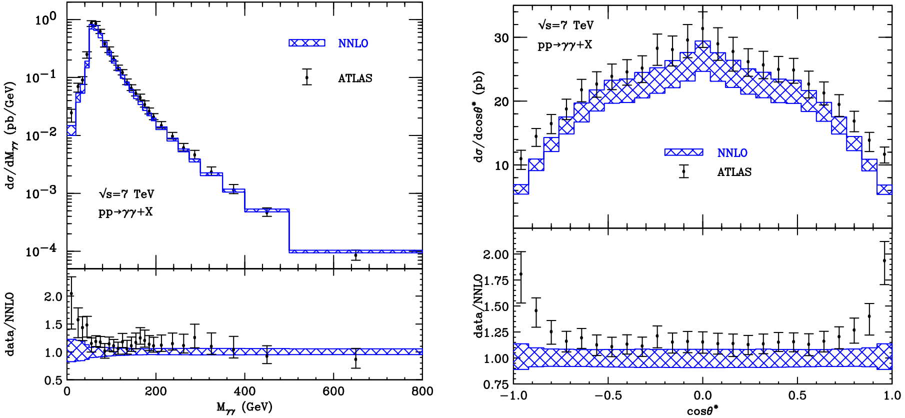
The differential cross section $d\sigma/dM_{\gamma\gamma}$. LO, NLO and NNLO results with scale dependence and NNLO K factor with scale dependence (left). Decomposition in the contributions of different initial-state partonic channels: $q\bar{q}$, qg, gg and the box $gg \to \gamma\gamma$ (right).



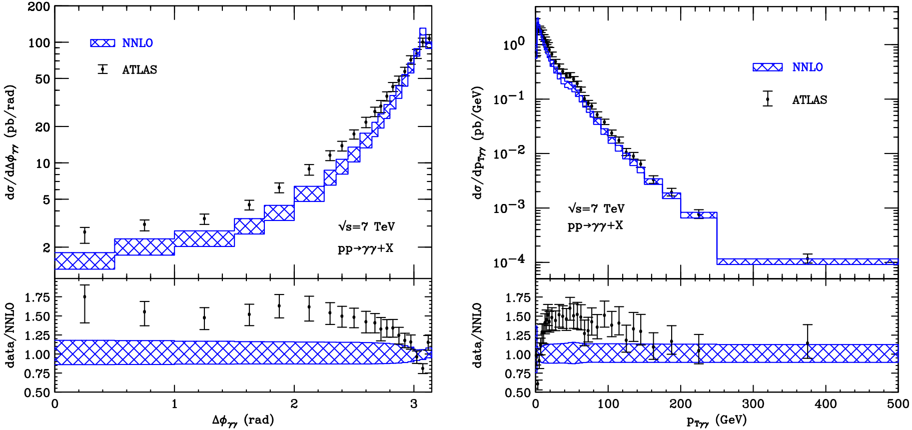
The differential cross section $d\sigma/d\cos\theta^*$.. LO, NLO and NNLO results with scale dependence and NNLO K factor with scale dependence (left). Decomposition in the contributions of different initial-state partonic channels: $q\bar{q}$, qg, gg and the box $gg \to \gamma\gamma$ (right).



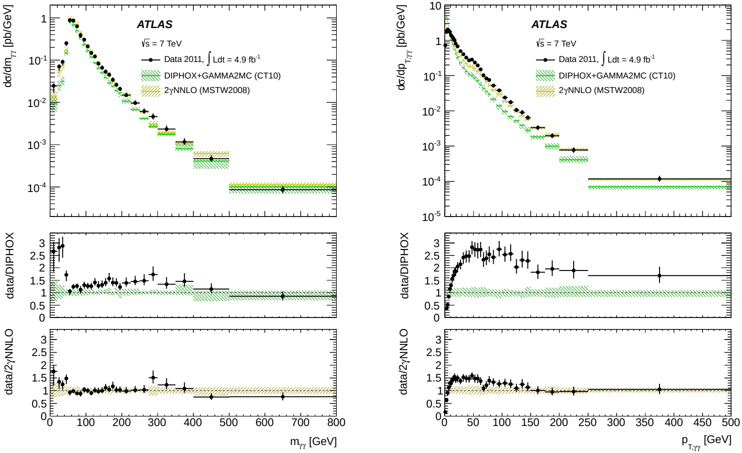
The differential cross sections $d\sigma/d\Delta\Phi_{\gamma\gamma}$ (left) and $d\sigma/dp_{T\gamma\gamma}$ (right). LO, NLO and NNLO results with scale dependence and NNLO K factor with scale dependence.



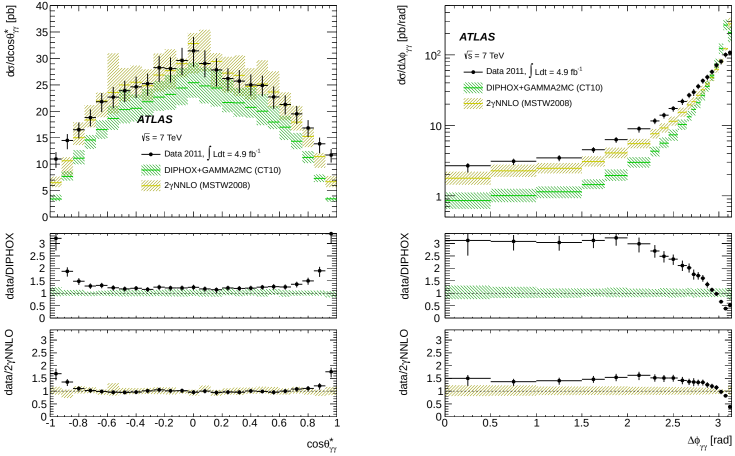
Comparison between ATLAS data at $\sqrt{s} = 7$ TeV and NNLO results (with scale dependence) for $d\sigma/dM_{\gamma\gamma}$ (left) and $d\sigma/d\cos\theta^*$ (right). The NNLO results are corrected for hadronization and underlying event effects.



Comparison between ATLAS data at $\sqrt{s} = 7$ TeV and NNLO results (with scale dependence) for $d\sigma/d\Delta\Phi_{\gamma\gamma}$ (left) and $d\sigma/dp_{T\gamma\gamma}$ (right). The NNLO results are corrected for hadronization and underlying event effects.



Comparison between ATLAS data at $\sqrt{s} = 7$ TeV with NLO (DIPHOX+GAMMA2MC) and NNLO (2γNNLO) results (with scale dependence) for $d\sigma/dM\gamma\gamma$ (left) and $d\sigma/dp_{T\gamma\gamma}$ (right).

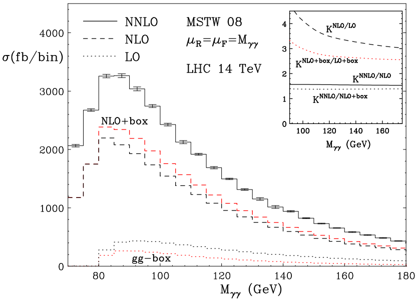


Comparison between ATLAS data at $\sqrt{s} = 7$ TeV with NLO (DIPHOX+GAMMA2MC) and NNLO (2γNNLO) results (with scale dependence) for $\cos\theta^*$ (left) and $d\sigma/\Delta\Phi_{\gamma\gamma}$ (right).

Conclusions

- Detailed study on differences between standard and smooth cone isolation up to NLO: results are consistent within the corresponding scale uncertainties.
- Smooth cone isolation: consistent theoretical framework for NNLO calculation.
- First calculation of full NNLO QCD corrections to direct Diphoton production in hadron collision using the q_T-subtraction formalism.
- Calculation included in a fully-exclusive public available parton-level Monte Carlo code: 2γNNLO.
- NNLO corrections found to be large: $\sim 50\%$ over NLO at the LHC.
- NNLO corrections essential away from the back-to-back region (effectively next-order corrections).
- NNLO uncertainty band: first reliable estimate of perturbative uncertainty in some region underestimate the *true* perturbative uncertainty.
- NNLO corrections clearly improves description of the LHC data.

Back-up



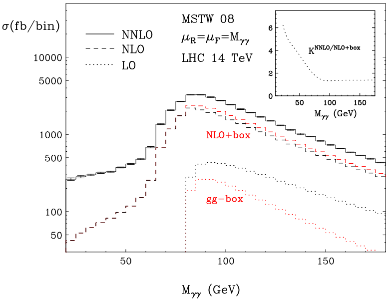
$M_{\gamma\gamma}$ spectrum at the LHC $\sqrt{s} = 14\ TeV$
Smooth cone isolation:
$\epsilon_\gamma = 0.5,\ n = 1,\ R = 0.4$
Scales: $\mu_R = \mu_F = M_{\gamma\gamma}$
Cuts:
$p_T^{\gamma,hard} > 40 GeV,\ p_T^{\gamma,soft} > 25 GeV,$
$|\eta^\gamma| < 2.5,\ 20 < M_{\gamma\gamma} < 250 GeV,$

- In the peak region:

$$\frac{\sigma^{NNLO}}{\sigma^{NLO}} \sim 1.55 \qquad \frac{\sigma^{NNLO}}{\sigma^{NLO+box}} \sim 1.35$$

NNLO corr. $\sim$ 55% of the total,
box contrib. $\sim$ 22% of NNLO,
$qg \sim$ 60% of NNLO.

- Large higher order corr. due to:
New large luminosity channels at each order (qg at NLO, gg at NNLO).
Asymmetric cuts:
new phase space region available beyond LO.
At LO $p_T^{\gamma,soft} = p_T^{\gamma,hard} > 40 GeV$,
$\Rightarrow M_{\gamma\gamma} > 80\ GeV$
Beyond LO $25 < p_T^{\gamma,soft} < 40 GeV$ and
$M_{\gamma\gamma} < 80\ GeV$ available.



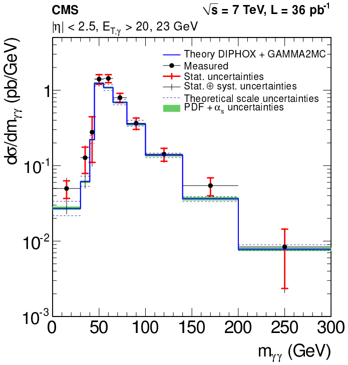
$M_{\gamma\gamma}$ spectrum at the LHC $\sqrt{s} = 14\ TeV$
Smooth cone isolation:
$\epsilon_\gamma = 0.5,\ n = 1,\ R = 0.4$
Scales: $\mu_R = \mu_F = M_{\gamma\gamma}$
Cuts:
$p_T^{\gamma,hard} > 40 GeV,\ p_T^{\gamma,soft} > 25 GeV,$
$|\eta^\gamma| < 2.5,\ 20 < M_{\gamma\gamma} < 250 GeV,$

- In the peak region:

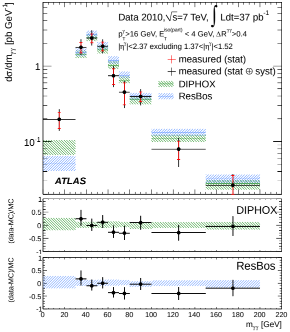
$$\frac{\sigma^{NNLO}}{\sigma^{NLO}} \sim 1.55 \qquad \frac{\sigma^{NNLO}}{\sigma^{NLO+box}} \sim 1.35$$

NNLO corr. $\sim$ 55% of the total,
box contrib. $\sim$ 22% of NNLO,
$qg \sim$ 60% of NNLO.

- Large higher order corr. due to:
New large luminosity channels at each order (qg at NLO, gg at NNLO).
Asymmetric cuts:
new phase space region available beyond LO.
At LO $p_T^{\gamma,soft} = p_T^{\gamma,hard} > 40 GeV$,
$\Rightarrow M_{\gamma\gamma} > 80\ GeV$
Beyond LO $25 < p_T^{\gamma,soft} < 40 GeV$ and $M_{\gamma\gamma} < 80\ GeV$ available.

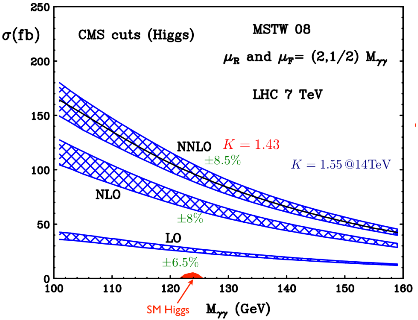


Invariant mass $M_{\gamma\gamma}$ spectrum measured by [CMS arXiv:1110.6461] compared with NLO QCD.



Invariant mass $M_{\gamma\gamma}$ spectrum measured by [ATLAS arXiv:1107.0581] compared with NLO QCD.

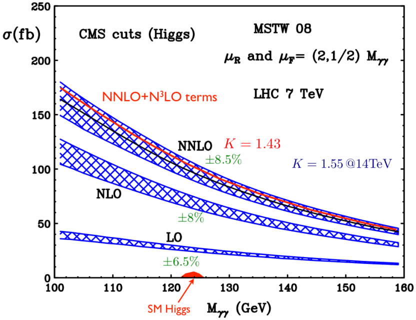
At LO photons are back-to-back: $M_{\gamma\gamma} \geq 2p_T^{\gamma,hard}$. For $M_{\gamma\gamma} \leq 2p_T^{\gamma,hard}$ the NLO is the lowest order result. NNLO corrections at low $M_{\gamma\gamma}$ are essential.



$M_{\gamma\gamma}$ spectrum at the LHC $\sqrt{s} = 7\ TeV$
Smooth cone isolation:
$\epsilon_\gamma = 0.05,\ n = 1,\ R = 0.4$
Scales: $M_{\gamma\gamma}/2 < \mu_R = \mu_F < 2M_{\gamma\gamma}$
Cuts:
$p_T^{\gamma,hard} > 40GeV,\ p_T^{\gamma,soft} > 30GeV,$
$|\eta^\gamma| < 2.5$ (excl. $1.44 < |\eta^\gamma| < 1.57$),
$100 < M_{\gamma\gamma} < 160GeV$.

- Naive LO and NLO scale variation bad estimate of perturbative uncertainty. Due to opening of new (large luminosity) channels.
- At NNLO all possible partonic channels are open: first reliable estimate of perturbative uncertainty.
- Some N^3LO terms (box corrections) are known [Bern,Dixon,Schmidt('02),gamma2MC]

Their effect ($\sim 5\%$) is contained in the NNLO band.



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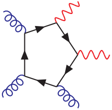

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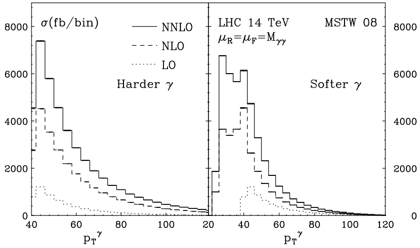


p_T spectrum of the harder and softer γ at the LHC $\sqrt{s} = 14\ TeV$

Smooth cone isolation:

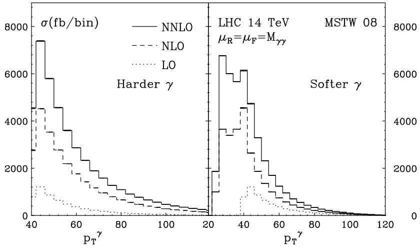
$\epsilon_\gamma = 0.5,\ n = 1,\ R = 0.4$

Scales: $\mu_R = \mu_F = M_{\gamma\gamma}$

Cuts:

$p_T^{\gamma,hard} > 40 GeV,\ p_T^{\gamma,soft} > 25 GeV,$
$|\eta^\gamma| < 2.5,\ 20 < M_{\gamma\gamma} < 250 GeV,$

- Asymmetric cuts: new phase space region available beyond LO. At LO $p_T^{\gamma,soft} = p_T^{\gamma,hard} > 40 GeV$ Beyond LO $25 < p_T^{\gamma,soft} < 40 GeV$ available: softer γ production enhanced $1/\tilde{p}^2 \log \tilde{p}^2$ when $\tilde{p} \equiv p_T^{\gamma,soft}/M_{\gamma\gamma} \ll 1$.
- Around LO kinematical boundary $p_T = 40\ GeV$, perturbative instabilities in $p_T^{\gamma,soft}$ distr.: Sudakov shoulder [Catani,Webber ('97)].
- For $p_T \gtrsim 50\ GeV$ small correction in $p_T^{\gamma,soft}$ distr. (both γ are hard).



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Scales: $\mu_R = \mu_F = M_{\gamma\gamma}$

Cuts:

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$|\eta^\gamma| < 2.5,\ 20 < M_{\gamma\gamma} < 250 GeV,$

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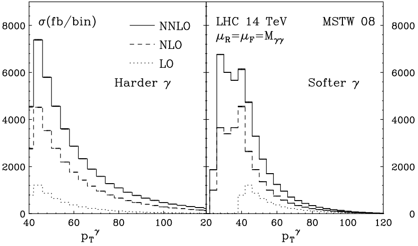

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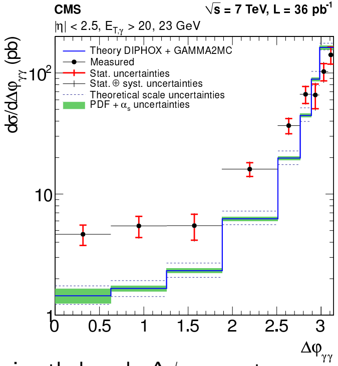
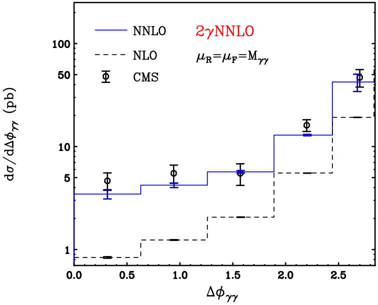
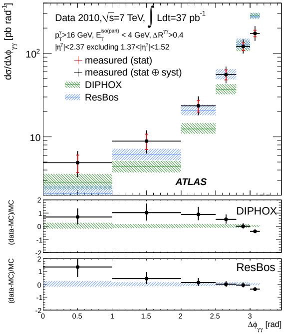


Azimuthal angle $\Delta\phi_{\gamma\gamma}$ spectrum measured by [CMS arXiv:1107.6461] compared with NLO QCD.



NLO and NNLO QCD corrections (CMS cuts but smooth cone isolation) compared with CMS data.

At LO photons are back-to-back: $\Delta\phi_{\gamma\gamma} = \pi$. For $\Delta\phi_{\gamma\gamma} < \pi$ the NLO is the lowest order result. NNLO corrections at low $\Delta\phi_{\gamma\gamma}$ are essential.



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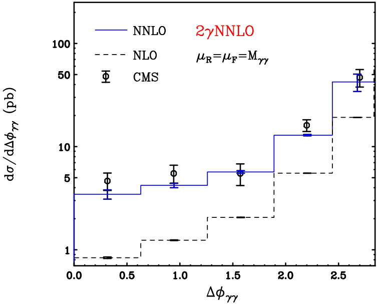

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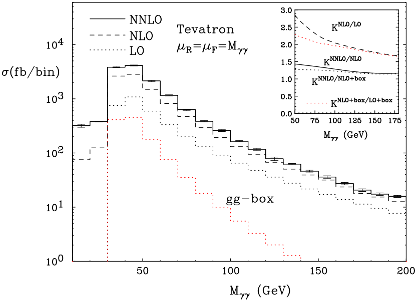
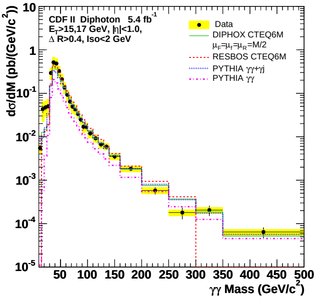


$M_{\gamma\gamma}$ spectrum at the Tevatron
$\sqrt{s} = 1.96\ TeV$
Smooth cone isolation:
$\epsilon_\gamma = 0.5,\ n = 1,\ R = 0.4$
Scales: $\mu_R = \mu_F = M_{\gamma\gamma}$
Cuts:
$p_T^{\gamma,hard} > 17GeV,\ p_T^{\gamma,soft} > 15GeV,$
$|\eta^\gamma| < 1,\ 10 < M_{\gamma\gamma} < 200GeV,$

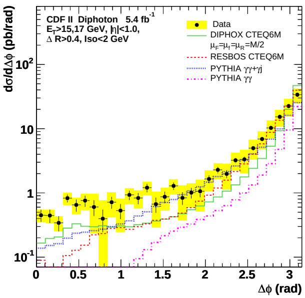
- In the peak region:

$$\frac{\sigma^{NNLO}}{\sigma^{NLO}} \sim 1.4 \qquad \frac{\sigma^{NNLO}}{\sigma^{NLO+box}} \sim 1.3$$

- Higher orders corrections smaller than at the LHC:
  Cuts only slightly asymmetric.
- For $M_{\gamma\gamma} > 80\ GeV$ box contribution smaller than at the LHC (probed higher value of parton momentum fractions).
- NNLO corrections still quite large ($\sim 30\%$).



Invariant mass $M_{\gamma\gamma}$ spectrum measured by [CDF arXiv:1106.5131] compared with NLO QCD.



Azimuthal angle $\Delta\phi_{\gamma\gamma}$ spectrum measured by [CDF arXiv:1106.5131] compared with NLO QCD.

Analogous discrepancy for low $M_{\gamma\gamma}$ and low $\Delta\phi_{\gamma\gamma}$ (away from back-to-back region) between CDF data and NLO QCD.

$\mathbf{q_T}$ resummation: $q\bar{q}$-annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \to F$.

$$\frac{d\sigma_F^{(res)}(p_1,p_2;\mathbf{q_T},M,y,\Omega)}{d^2\mathbf{q_T}\,dM^2\,dy\,d\Omega} = \frac{M^2}{s}\sum_{c=q,\bar{q}}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_q(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\;\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\;,$$

[Collins,Soper,Sterman('85)],
[Catani,de Florian,Grazzini('01)]

$b_0=2e^{-\gamma_E}\ (\gamma_E=0.57\ldots)$, $\quad x_{1,2}=\frac{M}{\sqrt{s}}\,e^{\pm y}$, $\quad L\equiv\ln Mb$

$$S_q(M,b)=\exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_q(\alpha_S(q^2))\;\ln\frac{M^2}{q^2}+B_q(\alpha_S(q^2))\right]\right\}\quad.$$

$$\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2}=H_q^F(x_1p_1,x_2p_2;\Omega;\alpha_S(M^2))\;C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\;C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\;,$$

$$A_q(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)},\quad B_q(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S)=1+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)},\qquad C_{qa}(z;\alpha_S)=\delta_{qa}\;\delta(1-z)+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n C_{qa}^{(n)}(z)\,.$$

LL($\sim\alpha_S^nL^{n+1}$): $A_q^{(1)}$; NLL($\sim\alpha_S^nL^n$): $A_q^{(2)}, B_q^{(1)}, H_q^{F\,(1)}, C_{qa}^{(1)}$; NNLL($\sim\alpha_S^nL^{n-1}$): $A_q^{(3)}, B_q^{(2)}, H_q^{F\,(2)}, C_{qa}^{(2)}$

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[Collins,Soper,Sterman('85)],

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$, $L \equiv \ln Mb$ [Catani,de Florian,Grazzini('01)]

$$S_q(M,b) = \exp\left\{-\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\left[A_q(\alpha_S(q^2))\,\ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2))\right]\right\} \;.$$

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$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $x_{1,2} = \frac{M}{\sqrt{s}}\; e^{\pm y}$, $L \equiv \ln Mb$

$$S_q(M,b) = \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \;\ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2)) \right] \right\} \;.$$

$$\left[H^F C_1 C_2 \right]_{q\bar{q};a_1 a_2} = H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))\;\; C_{q a_1}(z_1; \alpha_S(b_0^2/b^2))\;\; C_{\bar{q}\, a_2}(z_2; \alpha_S(b_0^2/b^2)) \;,$$

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$\mathbf{q_T}$ resummation: $q\bar{q}$-annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \to F$.

$$\frac{d\sigma_F^{(res)}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q}}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_q(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\;,$$

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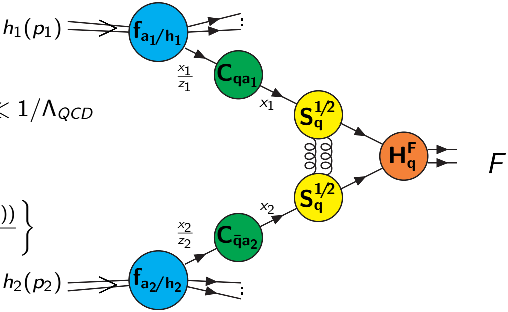
Transverse-momentum resummation formula



$$M \gg \Lambda_{QCD}\ , \quad b \gg 1/M\ , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2)) \times \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{s} \left[d\sigma^{(0)}_{q\bar{q},F} \right] H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$

$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1; \alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2; \alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\, C_{q_f\,a}(z; \alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

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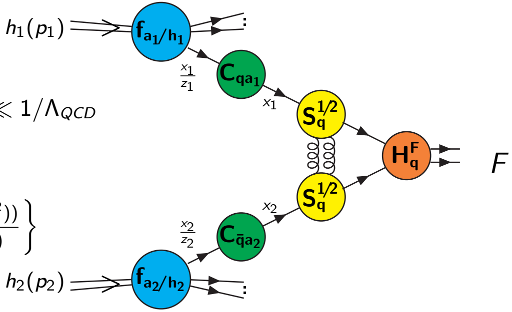

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Hard-collinear coefficients at NNLO

- Resummation coefficients in Sudakov form factor known since some time up to $\mathcal{O}(\alpha_S^2)$ ($A_c^{(1,2)}$, $B_c^{(1,2)}$), $A_c^{(3)}$ calculated more recently [Becher,Neubert('11)]
- Explicit NNLO *analytic* calculations of the q_T cross section (at small-q_T): *(i)* SM Higgs boson production [Catani,Grazzini('07,'12)] and *(ii)* DY process [Catani,Cieri,de Florian,G.F.,Grazzini('09,'12)].
- These calculations provide complete knowledge of the process-independent *collinear* coeff. $C_{ca}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ ($G_{ga}(z,\alpha_S)$ up to $\mathcal{O}(\alpha_S)$), and of the *hard-virtual* factor $H_c^F(\alpha_S)$ up to $\mathcal{O}(\alpha_S^2)$ for DY/H processes. In the *hard scheme*:

$$C_{qq}^{(1)}(z) = \frac{C_F}{2}(1-z)\,,\; C_{gq}^{(1)}(z) = \frac{C_F}{2}z\,,\; C_{qg}^{(1)}(z) = \frac{z}{2}(1-z)\,,$$

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$$H_q^{DY(1)} = C_F\left(\frac{\pi^2}{2}-4\right)\,,\; H_g^{H(1)} = C_A\pi^2/2 + \frac{11}{2}\,.$$

Analogous (bit longer) expressions for : $C_{qq}^{(2)}(z)\,, C_{qg}^{(2)}(z)\,, C_{gg}^{(2)}(z)\,, C_{gq}^{(2)}(z)\,, H_q^{DY(2)}\,, H_g^{H(2)}\,.$

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Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}),$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon),$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

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hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2},$$

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IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left[1 - \tilde{I}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) \,,$$

hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2} \,,$$

(α_S^k is the overall α_S power (e.g. $k = 0$ for DY, $k = 1$ for $gg \to H$)).

Hard factors at NNLO

- The previous all-order factorization formula was explicitly evaluated up to NNLO: we know the explicit expression of the *universal* subtraction operators up to two-loops $\tilde{I}_c^{(1)}(\epsilon)$, $\tilde{I}_c^{(2)}(\epsilon)$.
- We can straightforward apply the factorization formula to determine the NNLO hard-virtual factors from the knowledge of the two-loops amplitudes.
- E.g. diphoton production: we rederived the result for $H_q^{\gamma\gamma\,(1)}$ [Balazs et al.('98)] and (using the two-loop amplitudes [Anastasiou et al.('02)]) we obtained the $H_q^{\gamma\gamma\,(2)}$ [Catani,Cieri,de Florian,GF,Grazzini('12)]

$$H_q^{\gamma\gamma(1)} = \frac{C_F}{2}\left\{(\pi^2-7) + \frac{\left((1-v)^2+1\right)\ln^2(1-v) + v(v+2)\ln(1-v) + (v^2+1)\ln^2 v + (1-v)(3-v)\ln v}{(1-v)^2+v^2}\right\}.$$

$$H_q^{\gamma\gamma(2)} = \frac{1}{4\mathcal{A}_{LO}}\left[\mathcal{F}^{0\times2}_{inite,q\bar{q}\gamma\gamma;s} + \mathcal{F}^{1\times1}_{inite,q\bar{q}\gamma\gamma;s}\right] + 3\zeta_2\, C_F H_q^{\gamma\gamma(1)} - \frac{45}{4}\zeta_4 C_F^2 + C_F N_f\left(-\frac{41}{162} - \frac{97}{72}\zeta_2 + \frac{17}{72}\zeta_3\right)$$
$$+ C_F C_A\left(\frac{607}{324} + \frac{1181}{144}\zeta_2 - \frac{187}{144}\zeta_3 - \frac{105}{32}\zeta_4\right), \quad \text{where} \quad v = -(p_q - p_\gamma)^2/M^2.$$

- Analogous results were obtained for $ZZ, W\gamma, Z\gamma$ [Grazzini et al.('14)], [Cascioli et al.('14)],[Gehrmann et al.('14)] and $b\bar{b} \to H$ production [Harlander et al.('14)].

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