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Gravitational lensing by exotic objects

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Main reference:

HA, Mod. Phys. Lett. A, 32, 1730031 (2017) for a brief review on “GL by exotic objects”

See also a book “Lorentzian Wormholes” (Matt Visser)
Outline

1. Black Holes (BHs) and Wormholes (WHs)
2. Gravitational Lensing by BHs
3. Gravitational Lensing by WHs
4. Gravitational lensing by an exotic object:
   --- as a probe of new physics
1. Black Holes (BHs) and Wormholes (WHs)

BHs (candidates) are real and astrophysical objects by EM(X-ray) and GW methods.
BHs

Horizon is caused by strong gravity (pull)

Incoming only

Singularity
(New Physics?)

No outgoing
Wormholes (WHs) coined by John Wheeler (1957)

a spacetime “tunnel”

Speculative objects connecting two different points in a spacetime

non-trivial topology (non-simply connected)
WHs require a violation of some energy conditions (in GR) or a modification of gravity theories.

e.g.

**Null Energy Condition (NEC):**

For any null vector, \[ T_{\mu\nu} k^\mu k^\nu \geq 0 \]

**Averaged Null Energy Condition (ANEC):**

On a null curve, \[ \int_{\Gamma} T_{\mu\nu} k^\mu k^\nu d\lambda \geq 0 \]
On a construction of WHs (spacetime tunnels):

“Tidal fields” (in GR) are described by Raychaudhuri equation.

Amal Kumar Raychaudhuri (1923-2005)
Raychaudhuri equation for null geodesics

\[
\frac{d\hat{\theta}}{d\lambda} = -R_{\mu\nu} k^\mu k^\nu - 2\hat{\sigma}^2 - \frac{1}{2} \hat{\theta}^2 + 2\hat{\omega}^2
\]

\[\hat{\theta} \equiv P^\nu_\mu (\nabla_\nu k^\mu)\]

Expansion rate of null geodesic bundle

\[\hat{\sigma}_{\mu\nu} \equiv P^\alpha_\langle\mu P^\beta_\nu\rangle \nabla_\alpha k_{\beta}\]

Shear of the bundle

\[\hat{\omega}_{\mu\nu} \equiv P^\alpha_\langle\mu P^\beta_\nu\rangle \nabla_\alpha k_{\beta}\]

Vorticity of the bundle

\[
(P^\alpha_\mu \quad \text{Projection tensor})
\]
We consider the throat of a spherically symmetric WH.

For purely radial null geodesics,

\[ \hat{\sigma}_{\mu\nu} = 0 \quad \hat{\omega}_{\mu\nu} = 0 \]

Then,

\[ \frac{d\hat{\theta}}{d\lambda} = - R_{\mu\nu} k^\mu k^\nu - \frac{1}{2} \hat{\theta}^2 \]

The minimal cross-section area of the null bundle occurs at the WH throat:

\[ \hat{\theta} = 0 \quad \& \quad \frac{d\hat{\theta}}{d\lambda} \geq 0 \]
Therefore, at the throat

\[ R_{\mu\nu} k^\mu k^\nu \leq 0 \]

By using Einstein equation (in GR), this is rewritten as

\[ T_{\mu\nu} k^\mu k^\nu \leq 0 \]

Null energy condition must be violated at the throat.

N.B.

By integrating along the null geodesic, one can see that averaged null energy condition must be violated in WHs.
2. Gravitational Lensing by BHs (and stars)
Gravitational Microlensing

Light deflection

\[
\frac{4GM}{c^2b}
\]

Stronger gravitational pull

More light rays

Brighter image

Microlensing by WHs?
3. Gravitational Lensing by WHs
Light propagation in Ellis wormhole, especially deflection angle of light, is discussed by many authors.


Deflection angle of light in an Ellis wormhole geometry

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(Received 1 March 2012; published 3 May 2012)

Dey and Sen (2008) formula

\[ \alpha = \pi \left\{ \sqrt{\frac{2(r_0^2 + a^2)}{2r_0^2 + a^2}} - 1 \right\}, \]

The correct one is derived as

(Perlick, Gibbons and Vyska, Nakajima and HA)

\[ \alpha(b) = 2 \int_0^1 \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 t^2)}} - \pi = 2K(k) - \pi, \]

(9)

where \( t = b/R \) and \( k = a/b \). The integral in Eq. (9) is a complete elliptic integral of the first kind \( K(k) \), which admits a series expansion for \( k < 1 \). Hence, Eq. (9) is expanded as

\[ \alpha(b) = \pi \sum_{n=1}^{\infty} \left[ \frac{(2n - 1)!!}{(2n)!!} \right]^2 k^{2n}. \]

(10)
The application of Weierstrass elliptic functions to Schwarzschild null geodesics

G W Gibbons\(^1,2\) and M Vyska\(^2\)

\[ \frac{\delta \phi}{\pi} = -\frac{1}{4} \mu^2 - \frac{1}{2} \mu^3 - \frac{41}{64} \mu^4 - \frac{9}{16} \mu^5 - \frac{25}{256} \mu^6 + \frac{37}{128} \mu^7 + \frac{11959}{16384} \mu^8 + \frac{1591}{2048} \mu^9 \]
\[ + \frac{13311}{65536} \mu^{10} - \frac{29477}{32768} \mu^{11} - \ldots. \]  

(139)
Model

Ellis WH(1973)

\[ ds^2 = dt^2 - dr^2 - (r^2 + a^2)(d\theta^2 + \sin^2(\theta)d\phi^2), \]  
\[ \text{throat radius} \]

Deflection angle of light in weak field

\[ \alpha(r) \to \frac{\pi a^2}{4r^2} - \frac{5\pi a^4}{32r^4} + o \left( \frac{a}{r} \right)^6. \]  
\[ \left( \frac{4M}{r} \right) \text{ for BHs} \]
What is an astronomical implication by the different deflection of light in WHs?

GRavitational Microlensing by the Ellis Wormhole

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ABSTRACT

A method to calculate light curves of the gravitational microlensing of the Ellis wormhole is derived in the weak-field limit. In this limit, lensing by the wormhole produces one image outside the Einstein ring and another image inside. The weak-field hypothesis is a good approximation in Galactic lensing if the throat radius is less than $10^{11}$ km. The light curves calculated have gutters of approximately 4% immediately outside the Einstein ring crossing times. The magnification of the Ellis wormhole lensing is generally less than that of Schwarzschild lensing. The optical depths and event rates are calculated for the Galactic bulge and Large Magellanic Cloud fields according to bound and unbound hypotheses. If the wormholes have throat radii between 100 and $10^7$ km, are bound to the galaxy, and have a number density that is approximately that of ordinary stars, detection can be achieved by reanalyzing past data. If the wormholes are unbound, detection using past data is impossible.
Schwarzschild lensing can be achieved. Equations (A color version of this figure is available in the online journal.)

Figure 2.

The Astrophysical Journal

et al. 1992

Nemiroff & Wickramasinghe 1994

In conventional gravitational lensing theory (Schneider et al., 1992), the centroid position of the light distribution of a gravitationally lensed source is fixed in the sky. Astrometry gives a method for breaking this degeneracy as discussed later.

In making numerical figures, we employ (wormhole) in the sky. All quantities are normalized by the angular Einstein radius at y = 0.

For each extra dimension of the metric space, the centroid position by the Ellis wormhole and that by the Schwarzschild one. This is because the asymptotic behavior of the centroid displacement is different for wormholes. Following (Safonova et al., 1999), the centroid position of a source near a wormhole is essentially independent of the orientation and the transverse velocity (Alcock et al., 1986). Figure 2 shows the relative displacement trajectory by the Schwarzschild lens is an ellipse but slightly asymmetric along the x-axis like a tree leaf, par-
Figure 3. Sketch of the relation between the source trajectory and the lens (wormhole) in the sky. All quantities are normalized by the angular Einstein radius $\theta_E$.

In conventional gravitational lensing theory (Schneider et al. 1992), the convergence of light is expressed by a convolution of the surface mass density. Thus, we need to introduce negative mass to describe divergent lensing by the Ellis wormhole. However, negative mass is not a physical entity. As the lensing by the Ellis wormhole is convergent at the center, lensing at some other place must be divergent because the wormhole has zero asymptotic mass. For $\hat{\beta}_0 > 1$, the light curve of the wormhole has a basin at $t_0$ and no peak. Using these features, discrimination from Schwarzschild lensing can be achieved. Equations (7) and (20) indicate that the physical parameters ($D_L$, $a$, and $v_T$) are degenerate in $t_E$ and cannot be derived by fitting the light-curve data. This situation is the same as that for Schwarzschild lensing. To obtain or constrain these values, observations of the finite-source effect (Nemiroff & Wickramasinghe 1994) or parallax (Alcock et al. 1995) are necessary.

The detectability of the magnification of the star brightness depends on the timescale. The Einstein radius crossing time $t_E$ depends on the transverse velocity $v_T$. There is no reliable estimate of $v_T$ for wormholes. Here we assume that the velocity of the wormhole is approximately equal to the rotation velocity of stars ($v_T = 220 \text{ km s}^{-1}$) if it is bound to our Galaxy. If the wormhole is not bound to our Galaxy, the transverse velocity would be much higher. We assume $v_T = 5000 \text{ km s}^{-1}$ (Safonova et al. 2002) for the unbound wormhole. Table 2 shows the Einstein radius crossing times of the Ellis wormhole lensings for the Galactic bulge and LMC in both bound and

Figure 4. Light curves for $\hat{\beta}_0 = 0.2$ (top left), $\hat{\beta}_0 = 0.5$ (top right), $\hat{\beta}_0 = 1.0$ (bottom left), and $\hat{\beta}_0 = 1.5$ (bottom right). Thick red lines are the light curves for wormholes. Thin green lines are corresponding light curves for Schwarzschild lenses.

(A color version of this figure is available in the online journal.)
Does this demagnification conclude wormholes?
To answer this, Kitamura, Nakajima, HA (2013) we introduced a one-parameter model of a weak-field metric

\[ ds^2 = -\left(1 - \frac{\varepsilon_1}{r^n}\right)dt^2 + \left(1 + \frac{\varepsilon_2}{r^n}\right)dr^2 \]

\[ + r^2(d\theta^2 + \sin^2\theta d\phi^2) + O(\varepsilon_1^2, \varepsilon_2^2, \varepsilon_1 \varepsilon_2), \]

1. static and asymptotically flat
2. only in the weak field
3. n=1 : Schwarzschild metric
   n=2 : Ellis Worm Hole (EWH)
4. n>1 : zero ADM mass (massless)
After a conformal transformation,

\[
d\bar{s}^2 = -dt^2 + \left(1 + \frac{\varepsilon}{R^n}\right)dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2)
\]

+ \(O(\varepsilon^2)\),

where \(\varepsilon \equiv n\varepsilon_1 + \varepsilon_2\) and

\[
R^2 \equiv \frac{r^2}{(1 - \frac{\varepsilon_1}{r^n})}.
\]
Deflection angle of light is calculated in the textbook manner as

\[ \alpha = 2 \int_{R_0}^{\infty} \frac{d\phi(R)}{dR} dR - \pi = \frac{\varepsilon}{b^n} \int_0^{\frac{\pi}{2}} \cos^n \psi d\psi + O(\varepsilon^2), \]

\[ \int_0^{\frac{\pi}{2}} \cos^n \psi d\psi = \frac{(n - 1)!!}{n!!} \frac{\pi}{2} \text{ (even } n), \]

\[ = \frac{(n - 1)!!}{n!!} \text{ (odd } n), \]

\[ = \frac{\sqrt{\pi}}{2} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)} \text{ (real } n > 0). \]

always **positive** constant
Deflection angle of light is written as

$$\alpha(b) = \frac{\bar{\epsilon}}{b^n}$$

n=0 : Singular Isothermal Sphere (SIS)
n=1 : Stars and BHs
n=2 : Ellis Worm Hole (EWH)

This one-parameter model is used by Tsukamoto and Harada (2012, 2013).
How exotic is this lens model?

In the standard lens theory (in GR), convergence (surface mass density) is

\[ \kappa(b) = \frac{\tilde{\epsilon}(1 - n)}{2} \frac{1}{b^{n+1}} \]

If \( \epsilon > 0 \) and \( n > 1 \), negative convergence (divergent “lens”)

Analogy to optical lenses ---

convex lens  concave lens

\[ \kappa > 0 \]  \[ \kappa < 0 \]

“Standard” Grav Lens  “Diversing” unusual
<table>
<thead>
<tr>
<th></th>
<th>$\kappa &gt; 0$</th>
<th>$\kappa = 0$</th>
<th>$\kappa &lt; 0$</th>
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</thead>
<tbody>
<tr>
<td><strong>Non-vac. Ricci-focusing</strong></td>
<td>$\varepsilon &gt; 0 &amp; n &lt; 1$</td>
<td>$\varepsilon &lt; 0 &amp; n &gt; 1$</td>
<td>$\varepsilon &lt; 0 &amp; n &lt; 1$</td>
</tr>
<tr>
<td><strong>Vac. Weyl-focusing</strong></td>
<td>$\varepsilon &gt; 0 &amp; n &gt; 1$</td>
<td>$n = 1$</td>
<td>$\varepsilon &gt; 0 &amp; n &gt; 1$</td>
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<tr>
<td><strong>Non-vac. Ricci-defocusing</strong></td>
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<td></td>
<td>$\varepsilon &lt; 0 &amp; n &lt; 1$</td>
</tr>
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</table>
Lens Equation with thin lens approx.

$$\beta = \frac{b}{D_L} - \frac{D_{LS}}{D_S} \alpha(b),$$

$$\beta = \text{source angular position}$$
If $\varepsilon < 0$, repulsive like a concave lens

FIG. 3 (color online). Repulsive lens model ($\varepsilon < 0$). Solid curves denote $1/\hat{\theta}^n$ and straight lines mean $\hat{\theta} - \hat{\beta}$. Their intersections correspond to image positions that are roots for the lens equation. There are three cases: No image for a small $\hat{\beta}$ (dot-dashed line), a single image for a particular $\hat{\beta}$ (dotted line), and two images for a large $\hat{\beta}$ (dashed line). The two images are on the same side of the lens object.
For $\varepsilon > 0$,

Einstein ring for $\beta = 0$

$$\theta_E \equiv \left( \frac{\bar{\varepsilon} D_{LS}}{D_SD_L^n} \right)^{\frac{1}{n+1}}.$$ 

If $\varepsilon < 0$,

(tentative) Einstein ring radius

$$\theta_E \equiv \left( \frac{|\bar{\varepsilon}| D_{LS}}{D_SD_L^n} \right)^{\frac{1}{n+1}}.$$
Three typical observables in GL

1) Image **brightness** (micro-lens)

2) Image **shape** (macro-lens)

3) Image **motion** (micro-lens)
1) Image brightness(micro)

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Demagnifying gravitational lenses toward hunting a clue of exotic matter and energy

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\[ \hat{\beta} \equiv \beta / \theta_E \text{ and } \hat{\theta} \equiv \theta / \theta_E \]

\[ \hat{\beta} = \hat{\theta} - \frac{1}{\hat{\theta}^n} \quad (\hat{\theta} > 0), \]

\[ \hat{\beta} = \hat{\theta} + \frac{1}{(-\hat{\theta})^n} \quad (\hat{\theta} < 0), \]
For small beta (= source close to lens)

\[ \hat{\theta}_+ = 1 + \frac{1}{n + 1} \hat{\beta} + \frac{1}{2} \frac{n}{(n + 1)^2} \hat{\beta}^2 + O(\hat{\beta}^3) \quad (\hat{\theta} > 0), \]

\[ \hat{\theta}_- = -1 + \frac{1}{n + 1} \hat{\beta} - \frac{1}{2} \frac{n}{(n + 1)^2} \hat{\beta}^2 + O(\hat{\beta}^3) \quad (\hat{\theta} < 0). \]

Axisymmetric along line of sight

\[ A \text{ is } |(\beta/\theta) \times (d\beta/d\theta)|^{-1}. \]

\[ A_{\pm} = \frac{1}{\hat{\beta}(n + 1)} + O(\hat{\beta}^0), \]
\[ A_{\text{tot}} \equiv A_+ + A_- = \frac{2}{\hat{\beta}(n + 1)} + O(\hat{\beta}^0). \] (12)

Total demagnification \((A_{\text{tot}} < 1)\), if

\[ \hat{\beta} > \frac{2}{n + 1} \]

under weak field, thin lens and small \(\beta\) approximations.

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only one intersection of the two lines that corresponds to one image position. Similarly, only one image appears for \( C_1^8 < 0 \).

For a general positive \( n \) (e.g., \( n = 5 \)), it is impossible to find exact solutions for the modified lens equation. To clarify the parameter dependence, we employ analytic but approximate methods rather than numerical calculations. Furthermore, in astronomy only the significantly amplified images become detectable in gravitational microlensing. Such events occur only when a source such as a distant star crosses the Einstein ring. We thus focus on such an Einstein ring-crossing case as \( C_1^2 < 1 \) in units of the Einstein ring, for which Eqs. (7) and (8) are solved in the Taylor series form with respect to \( C_1^2 \). We obtain

\[
\hat{\beta} = 0.187
\]

Furthermore, for the Schwarzschild case \( n = 1 \), \( \hat{\beta} = 0.187 \). This is always larger than unity for \( \hat{\beta} < 1 \), in concordance with the well-known fact. Demagnification of the total lensed images could occur, however, if \( \hat{\beta} > 2n+1 \).
The larger the power $n$, the more likely the demagnification. One might guess that demagnification could be caused for a smaller $\tilde{\mathcal{C}}_{12}$, especially $\tilde{\mathcal{C}}_{12} = 0$. However, this is not the case. Equation (13) suggests that the total demagnification could occur only when $\tilde{\mathcal{C}}_{12}$ is small but larger than the critical value $\tilde{\mathcal{C}}_{12} = \left(\frac{n+1}{n}\right)^{\frac{1}{n-1}}$ under a large-$n$ approximation.

Note that the compatibility of the assumption $\tilde{\mathcal{C}}_{12} < 1$ and Eq. (13) implies $n > 1$. Namely, Eq. (13) becomes a better approximation as $n$ grows larger than unity.

The above argument is based on the near-zone approximation ($\tilde{\mathcal{C}}_{12} < 1$). For a test of the analytic result, we perform numerical calculations. We consider $n = 10$, which might be one of the higher-dimensional models inspired by string theory. Equation (13) suggests that demagnification of the total lensed images could occur only for $\tilde{\mathcal{C}}_{12} > \tilde{\mathcal{C}}_{12} = \left(\frac{n+1}{n}\right)^{\frac{1}{n-1}} = 0.182$.

Figure 1 shows numerical results for $n = 1, 2, 3, 10$. In the case of $n = 10$, the analytic result for the critical value $\tilde{\mathcal{C}}_{12} = \left(\frac{n+1}{n}\right)^{\frac{1}{n-1}} = 0.182$ is in good agreement with the numerical one, $\tilde{\mathcal{C}}_{12} = 0.187$.

Figure 2 shows numerical light curves for $n = 1, 2, 3, 10$. As the power $n$ is larger, time-symmetric demagnification parts in the light curves become longer in time and larger in depth. Cases of $n = 3$ and 10 show maximally $\tilde{\mathcal{C}}_{12} = 0.1$ and $\tilde{\mathcal{C}}_{12} = 0.1$ percent depletion of the light, when the source position is $\tilde{\mathcal{C}}_{12} = \left(\frac{n+1}{n}\right)^{\frac{1}{n-1}}$.

Before closing this section, we briefly mention an effective mass. A simple application of the standard lens theory suggests that the deflection ($\tilde{\mathcal{C}}_{11} = \tilde{\mathcal{C}}_{22}$) and magnification studied here correspond to a convergence (scaled surface-mass density) of the form

$$\tilde{\mathcal{C}}_{20} \left(\frac{n}{b^n - 1}\right)^{\frac{1}{2n-1}}.$$

For $n > 1$, therefore, the effective surface-mass density of the lens object is interpreted as negative in the framework of the standard lens theory. This means that the matter (and energy) need to be exotic if $n > 1$.
Time-symmetric demagnification in light curves will be an evidence for EWH (n=2) but not a proof.
2) Image shape (macro)

Gravitational lensing shear by an exotic lens object with negative convergence or negative mass

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2D-vectorial equations

A. \( \varepsilon > 0 \) case

\[
\hat{\beta} = \hat{\theta} - \frac{\hat{\theta}}{\hat{\theta}^{n+1}} \quad (\hat{\theta} > 0),
\]

\[
\hat{\beta} = \hat{\theta} - \frac{\hat{\theta}}{(-\hat{\theta})^{n+1}} \quad (\hat{\theta} < 0),
\]

magnification matrix \( A_{ij} \equiv \partial \beta^i / \partial \theta_j \)

\[
(A_{i,j}) = \begin{pmatrix}
1 - \frac{1}{\hat{\theta}^{n+1}} + (n + 1) \frac{\hat{\theta}_x \hat{\theta}_x}{\hat{\theta}^{n+3}} & (n + 1) \frac{\hat{\theta}_x \hat{\theta}_y}{\hat{\theta}^{n+3}} \\
(n + 1) \frac{\hat{\theta}_x \hat{\theta}_y}{\hat{\theta}^{n+3}} & 1 - \frac{1}{\hat{\theta}^{n+1}} + (n + 1) \frac{\hat{\theta}_y \hat{\theta}_y}{\hat{\theta}^{n+3}}
\end{pmatrix}.
\]

for \( \hat{\theta} > 0 \)
Axisymmetry enables to diagonalise the magnification matrix as

\[
(A_{ij}) = \begin{pmatrix}
1 - \kappa - \gamma & 0 \\
0 & 1 - \kappa + \gamma
\end{pmatrix}
\equiv \begin{pmatrix}
\lambda_- & 0 \\
0 & \lambda_+
\end{pmatrix},
\]

\[
\lambda_+ = \frac{\hat{\beta}}{\hat{\theta}} = 1 - \frac{1}{\hat{\theta}^{n+1}},
\]

\[
\lambda_- = \frac{d\hat{\beta}}{d\hat{\theta}} = 1 + \frac{n}{\hat{\theta}^{n+1}}.
\]
convergence \hspace{1cm} \kappa = 1 - \frac{\lambda_+ + \lambda_-}{2} = \frac{1 - n}{2} \frac{1}{\hat{\theta}^{n+1}},

shear \hspace{1cm} \gamma = \frac{\lambda_+ - \lambda_-}{2} = - \frac{1 + n}{2} \frac{1}{\hat{\theta}^{n+1}},
\[ \lambda_- > \lambda_+ \]
B. $\varepsilon < 0$ case

repulsive

like a concave lens

$$\hat{\beta} = \hat{\theta} + \frac{\hat{\theta}}{\hat{\theta}^n + 1} \quad (\hat{\theta} > 0),$$

$$\hat{\beta} = \hat{\theta} + \frac{\hat{\theta}}{(-\hat{\theta})^n + 1} \quad (\hat{\theta} < 0).$$

FIG. 3 (color online). Repulsive lens model ($\varepsilon < 0$). Solid curves denote $1/\hat{\theta}^n$ and straight lines mean $\hat{\theta} - \hat{\beta}$. Their intersections correspond to image positions that are roots for the lens equation. There are three cases: No image for a small $\hat{\beta}$ (dot-dashed line), a single image for a particular $\hat{\beta}$ (dotted line), and two images for a large $\hat{\beta}$ (dashed line). The two images are on the same side of the lens object.
The convergence parameter case, two images appear on the same side with at most two positive roots. Figure 1 shows that there are (\epsilon < 0) lensing by a repulsive case that might correspond to the lensing by a spherical void might be negative. Therefore, cosmic voids in cosmological simulations, because the present model with larger than the cosmological perturbation approach based on the scalar perturbation and the density contrast in the flat (Minkowskian) background spacetime. If one focuses on the two image cases, from which the single image approach the particular value.

\[ \text{radially elongated} \]

\[ \lambda_- < \lambda_+ \]
Finally, we mention the dependence on the exponent $n$.

A significantly elongated case such as a giant arc appears near the Einstein ring ($\sum C_{18} C_{24} n$), around which Eqs. (11) and (12) are expanded as

$$\sum C_{21} + \frac{1}{C_0} = \left(n + 1\right) \left(\sum C_{18} C_0 n\right)$$

and

$$\sum C_{19} \frac{1}{C_0} \left(n + 1\right) = \frac{\left(n + 1\right)^2}{C_0}$$

where we used the identity $\sum C_{18} = 1 + \left(\sum C_{18} C_0 n\right)$. The ratio of the tangential elongation to the radial one (corresponding to the arc shape) is

$$\sum C_{21} + \frac{1}{C_0} = \frac{1}{C_0} \left(n + 1\right) \left(\sum C_{18} C_0 n\right) + O\left(\left(\sum C_{18} C_0 n\right)^2\right)$$

This suggests that, for the fixed observed lens position $\sum C_{18}$, elongation of images becomes weaker, when $n$ becomes larger. This dependence on $n$ is true of also the secondary image.

**B. “< 0 case**

Let us study the “< 0 case. In the units of the Einstein ring radius, Eq. (4) is rewritten in the vectorial form as

![Numerical images](image)

**Fig. 2 (color online).** Numerical figures of lensed images for attractive ($\varepsilon > 0$) and repulsive ($\varepsilon < 0$) cases. They are denoted by dashed curves. We take $n = 2$. The source for each case is denoted by solid circles, which are located on the horizontal axis and the vertical one for $\varepsilon < 0$ and $\varepsilon > 0$, respectively.
No lens

\[ M > 0 \]

\[ M < 0 \]

courtesy of Koji Izumi
3) Image motion (micro)

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Microlensed image centroid motions by an exotic lens object with negative convergence or negative mass

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(Received 25 July 2013; published 3 April 2014)
Observables in astrometry (e.g. Gaia and JASMINE) are

the centroid position of the light distribution

$$\hat{\theta}_{pc} = \frac{A_1 \hat{\theta}_1 + A_2 \hat{\theta}_2}{A_{tot}},$$

The relative displacement of the image centroid with respect to the source position

$$\delta \hat{\theta}_{pc} = \hat{\theta}_{pc} - \hat{\beta}.$$
Many works for BH and Binary Lens


EWH case

FIG. 2: Centroid motions as $(\hat{\theta}_{pc}, x, \hat{\theta}_{pc}, y)$ for $\varepsilon > 0$ (convex-type attractive models). The solid and dashed curves correspond to $\hat{\beta}_0 = 3$ and $\hat{\beta}_0 = 0$, respectively. The horizontal axis along the source linear motion is $\hat{\theta}_{pc}, x$ and the vertical axis is $\hat{\theta}_{pc}, y$. Top left: $n = 0.5$. Top right: $n = 1$. Bottom left: $n = 3$. Bottom right: $n = 10$.

Convex-type Star and BH Source motion
FIG. 3: Centroid shifts $\delta \hat{\theta}_{pc}$ for $\varepsilon > 0$ (convex-type attractive models). The solid and dashed curves correspond to $\hat{\beta}_0 = 3$ and $\hat{\beta}_0 = 0.3$, respectively. The horizontal axis along the source velocity is $\delta \hat{\theta}_{pc,x}$ and the vertical axis is $\delta \hat{\theta}_{pc,y}$. Top left: $n = 0.5$ Top right: $n = 1$. Bottom left: $n = 3$. Bottom right: $n = 10$. 
FIG. 6: Centroid motions as $(\hat{\theta}_{pc,x}, \hat{\theta}_{pc,y})$ for $\varepsilon < 0$ (repulsive models). The solid and dashed curves correspond to $\hat{\beta}_0 = 3$ and $\hat{\beta}_0 = 0.3$, respectively. The horizontal axis along the source linear motion is $\hat{\theta}_{pc,x}$ and the vertical axis is $\hat{\theta}_{pc,y}$. The dashed curves do not exist for small $\hat{\beta}$, where no images appear. Top left: $n = 0.5$ Top right: $n = 1$. Bottom left: $n = 3$. Bottom right: $n = 10$. 

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FIG. 7: Centroid shifts $\delta \hat{\theta}_{pc}$ for $\varepsilon < 0$ (concave-type repulsive models). The solid and dashed curves correspond to $\hat{\beta}_0 = 3$ and $\hat{\beta}_0 = 0.3$, respectively. The horizontal axis along the source velocity is $\delta \hat{\theta}_{pc,x}$ and the vertical axis is $\delta \hat{\theta}_{pc,y}$. The dashed curves are not closed, because no images appear for small $\hat{\beta}$. Top left: $n = 0.5$ Top right: $n = 1$. Bottom left: $n = 3$. Bottom right: $n = 10$. 
Observational bounds
OBSERVATIONAL UPPER BOUND ON THE COSMIC ABUNDANCES OF NEGATIVE-MASS COMPACT OBJECTS AND ELLIS WORMHOLES FROM THE SLOAN DIGITAL SKY SURVEY QUASAR LENS SEARCH

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ABSTRACT

The latest result in the Sloan Digital Sky Survey Quasar Lens Search (SQLS) has set the first cosmological constraints on negative-mass compact objects and Ellis wormholes. There are no multiple images lensed by the above two exotic objects for \( \sim 50,000 \) distant quasars in the SQLS data. Therefore, an upper bound is put on the cosmic abundances of these lenses. The number density of negative-mass compact objects is \( n < 10^{-8}(10^{-4}) \, h^3 \, \text{Mpc}^{-3} \) at the mass scale \( |M| > 10^{15}(10^{12}) \, M_\odot \), which corresponds to the cosmological density parameter \( |\Omega| < 10^{-4} \) at the galaxy and cluster mass range \( |M| = 10^{12-15} \, M_\odot \). The number density of the Ellis wormhole is \( n < 10^{-4} \, h^3 \, \text{Mpc}^{-3} \) for a range of the throat radius \( a = 10-10^4 \, \text{pc} \), which is much smaller than the Einstein ring radius.

Key words: cosmology: observations – gravitational lensing: strong
The horizontal axis is the absolute value of mass.

In order to evaluate the observational upper bound on the number density of the negative-mass compact objects, the vertical axis shows the upper bound on the cosmological number density divided by the cosmological critical density, \( \Omega \) is defined as the mass density divided by the cosmological critical density, \( \Omega = \frac{\rho}{\rho_c} \). As shown in the figure, the density parameter for the negative mass, \( \Omega_n < |\Omega| \), is less abundant than a star (\( \Omega \approx 10^{-4} \)). The upper-bound curves in Figure 2 show the upper bound for the Ellis wormhole. The horizontal axis is the throat radius of the lenses. For the galaxies and the galactic cluster with typical mass \( 10^{17} \) (orange), \( 10^{18} \) (blue), and \( 10^{19} \) (green). The black curve is the result without the magnification bias. Note that the probability is close to us since the quasar is lensed by the source redshift is set to be \( z_0 = 0.2 \). The source redshift is set to be \( z_0 = 0.2 \).
Conclusion

Wormholes and other exotic objects may be a probe of new physics, such as an exotic equation of state of matter/energy.

Brightness anomaly and so on in gravitational lens observations may provide a clue for exotic objects.

We discussed the inverse-power form of the spacetime metric as a phenomenological exotic lens model.
Thank you!
Backup files
Application to cosmology

**Cosmic voids:** effective $\kappa < 0$


“Measuring the mass distribution of voids with stacked weak lensing”

Numerical simulations for near-future surveys

“First measurement of gravitational lensing by cosmic voids in SDSS”, Melchior et al., ArXiv:1309.2045
Concluding remarks

Exotic lens models suggest unusual observational features.

They might be used for searching (or constraining) exotic matter/energy/gravity.

Dark matter and Dark energy play a role in cosmology.

Is there another (3rd) dark component in the universe?
TABLE II: Einstein radii and model parameters for Bulge and LMC lensings. $\theta_E$ is the angular Einstein radius, $R_E$ is the Einstein radius, and $\bar{\varepsilon}$ and $n$ are the two model parameters. $D_S = 8kpc$ and $D_L = 4kpc$ are assumed for Bulge. $D_S = 50kpc$ and $D_L = 25kpc$ are assumed for LMC.

<table>
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<th>$\theta_E (mas)$</th>
<th>$R_E (km)$</th>
<th>$\bar{\varepsilon}$</th>
<th>$R_E (km)$</th>
<th>$\bar{\varepsilon}$</th>
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</table>
TABLE III: Einstein radius crossing times for Bulge and LMC lensings. $t_E$ is the Einstein radius crossing time. $D_S = 8kpc$ and $D_L = 4kpc$ are assumed for Bulge. $D_S = 50kpc$ and $D_L = 25kpc$ are assumed for LMC. $v_T = 220km/s$ is assumed for Bulge and LMC. In this table, the Einstein radius is calculated by $R_E = v_T \times t_E$ from the definition of the Einstein radius crossing time. Here, the input is $t_E \sim 10^{-3} - 10^{3}(day)$, namely $1(min.) - 3(yr.)$.

<table>
<thead>
<tr>
<th>$t_E(day)$</th>
<th>$R_E(km)$</th>
<th>$\bar{\theta}/R_E$ [Bulge]</th>
<th>$\bar{\theta}/R_E$ [LMC]</th>
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<td>$5.0 \times 10^{-8}$</td>
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</table>
Raychaudhuri’s equation for null geodesics

\[ \dot{\theta} = -\frac{1}{2} \dot{\theta}^2 - 2 \dot{\sigma}^2 + 2 \dot{\omega}^2 - T_{\mu\nu} U^\mu U^\nu \]

\[ T_{\mu\nu} U^\mu U^\nu \geq 0 \]

**Null Energy Condition**

But, exotic objects (e.g. wormholes) may violate Null Energy Condition.

Ricci focusing may be **negative**, while Weyl focusing is **positive**.

[N.B., Wormholes without exotic matter in Einstein-Gauss-Bonnet-Dilaton gravity, Kanti+, PRL (2011)]