Dark matter as a complex scalar field: new cosmological constraints and detectability by LIGO

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Scalar Field Dark Matter: Cold Dark Matter Variant

Small-Scale Problems:
Discrepancies between dark matter simulations and observations on sub-halo scales

- Missing satellites
- Cuspy Core
- Too big to fail

Cold Dark Matter (CDM) candidates

- Standard CDM: WIMP, QCD Axion, etc.
- Scalar Field Dark Matter

Boylan-Kolchin et al. 2012
Motivation for **Scalar Field Dark Matter (SFDM):**
*alternative DM candidate to explain discrepancies*

**SFDM:** Ultra-light ($m \ll 10^{-5}$ eV), “cold” bosons ($T \simeq 0$) in an (effective) classical field description

**Astrophysics motiv’:**
- Suppression of DM clustering below a scale compatible with observations
- Suppression of central DM densities in galaxies to explain cored density profiles in DM-dominated (dwarf) galaxies
- Accomplish all this, even if we never detect DM

**Particle physics motiv’:**
- Ultra-light bosons in extensions of the SM, e.g.
  - (a multitude of) axion-like particles ("string axiverse"),
  - other extra-dimensional cosmologies (akin to KK modes),
  - pseudo-Nambu-Goldstone bosons (upon symmetry breaking in early Universe)
Particle Dark Matter (SFDM), aka Bose-Einstein Condensed Cold Dark Matter (BEC-CDM)

- Particles created with low entropy per particle → BEC → classical field Lagrangian for the DM condensate ("order parameter")
- Complex scalar field $\psi = |\psi| e^{i\theta}$ → U(1) symmetry: charge conservation

$$\mathcal{L} = \frac{\hbar^2}{2m} g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - V(\psi)$$

Units: $[\mathcal{L}] = [\text{eV/cm}^3]$, $[\psi] = \text{cm}^{-3/2}$, (+,-,-,-)

- Assume no coupling to the SM within this EFT description

- Choice of $V(\psi)$ and initial conditions determine
  i) the evolution of SFDM, hence the evolution of the background Universe
  ii) suppression of small-scale structure for $L < L_{SFDM}$ with $L_{SFDM} = \max \{\lambda_{\text{deB}}, l_{\text{SI}}\}$
Scalar Field Dark Matter (SFDM), aka Bose-Einstein Condensed Cold Dark Matter (BEC-CDM)

- **choice of potential V:**
  
  (rest-mass) quadratic term, \((mc^2/2)|\psi|^2\)
  
  \((\rightarrow \text{CDM-like in late Universe}),\)

  plus a possible repulsive self-interaction, \((\lambda/2)|\psi|^4\)
  
  \((\rightarrow \text{radiation-like in early Universe})\)

  \[
  \rightarrow \text{fundamental SFDM parameters: } m \text{ and } \lambda \quad \lambda = \hat{\lambda} \frac{\hbar^3}{m^2 c}
  \]

- **Initial condition:**

  conserved charge Q of SFDM determines energy density of DM today

  large-Q limit „spintessence“: \(\rho_{\text{SFDM,0}} = Qmc^2\)

  (a version of “asymmetric DM”)

  \[
  \rho_{\text{SFDM,0}} = n_{\text{SFDM,0}}mc^2 = \Omega_{\text{DM}}\rho_{\text{crit,0}}
  \]
Equations of motion

Klein-Gordon equation for the SFDM field $\psi$ ...

\[ g^{\mu\nu} \partial_\mu \partial_\nu \psi - g^{\mu\nu} \Gamma^\sigma_{\mu\nu} \partial_\sigma \psi + \frac{m^2 c^2}{\hbar^2} \psi + \frac{2\lambda m}{\hbar^2} |\psi|^2 \psi = 0 \]

...which evolves in a classical GR background

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]
Well into the matter-dominated era, they simplify to ...

nonlinear Schrödinger-Poisson system

\[
\frac{i\hbar}{\partial t} \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi + \lambda |\psi|^2 \psi + m\Phi \psi
\]

\[
\Delta \Phi = 4\pi G m |\psi|^2
\]
Numbers for SFDM parameters

„Typical“ numbers of models:

**QCD axion:** \( m \approx 6 \times 10^{-6} \text{ eV} \frac{10^{12} \text{ GeV}}{f} \) \( 10^9 \leq f \leq 10^{12} \text{ GeV} \)

(attractive) SI usually neglected: \( \lambda \sim 10^{-57} \)

**ultra-light axion-like particles:** \( m \sim (10^{-33} - 1) \text{ eV} \),

\( f \) unknown, but reasons for \( f \sim 10^{16} \text{ GeV} \)

SI usually neglected

**(other) bosons for DM:** \( m \sim (10^{-27} - 1) \text{ eV} \),

SI usually neglected, but we don't: (positive) \( \lambda \sim 10^{-93} - 10^{-83} \)

*For the latter two:* Jeans/virial scale can extend to galactic scales ~ kpc!
<table>
<thead>
<tr>
<th>Halo mass ([M_\odot])</th>
<th>Size [kpc]</th>
<th>Boson mass [eV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milky Way (MW)</td>
<td>10^{12}</td>
<td>100</td>
</tr>
<tr>
<td>Dwarf galaxy (DG)</td>
<td>10^{10}</td>
<td>10</td>
</tr>
<tr>
<td>Dwarf spheroidal (dSph)</td>
<td>10^{8}</td>
<td>1</td>
</tr>
<tr>
<td>Minihalo (MH)</td>
<td>10^{6}</td>
<td>0.1</td>
</tr>
</tbody>
</table>

TRD, Shapiro (1209.1835)

Jeans / virial scales bounded from below by

- **no self-interaction:** \( L_{\text{deB}} \leq R \rightarrow m \geq m_H \) (Heisenberg uncertainty “pressure”)

\[
m_H := \frac{\hbar}{R^2 (\pi G \bar{\rho})^{1/2}} = 1.066 \cdot 10^{-22} \left( \frac{R}{1 \text{ kpc}} \right)^{-1/2} \left( \frac{M}{10^8 M_\odot} \right)^{-1/2} \text{ eV}
\]

- or else gravity is balanced by a *positive* self-interaction coupling:

\[
L_{\text{deB}} \ll R \rightarrow m >> m_H \quad \text{and} \quad \lambda >> \lambda_H \quad \text{(self-interaction “pressure“)}
\]

\[
\lambda_H := \frac{\hbar^2}{2 \bar{\rho} R^2} = 2.252 \cdot 10^{-62} \left( \frac{R}{1 \text{ kpc}} \right) \left( \frac{M}{10^8 M_\odot} \right)^{-1} \text{ eV cm}^3
\]
\( \Lambda\)SFDM Model (2014) + GW (2016)

**2014:** take the same cosmic inventory as the basic \( \Lambda \)CDM model, except that CDM is replaced by SFDM → \( \Lambda \)SFDM (1310.6061, PRD 89, 083536 (2014))

**2016:** add stochastic GW background (SGWB) from inflation self-consistently to it (1611.07961)

Cosmological parameters from Planck 2013/2015 (assume SM neutrinos massless)

\[
\Omega_m = \Omega_b + \Omega_c
\]

\[\Omega_\Lambda = 1 - \Omega_m - \Omega_r\] (2014)

\[\Omega_\Lambda = 1 - \Omega_m - \Omega_r - \Omega_{GW}\] (2016)

- SFDM particle parameters: \( m, \lambda/(mc^2)^2 \)

\[
\lambda/(mc^2)^2 = 1 \times 10^{-18}\text{ eV}^{-1}\text{cm}^3 \Rightarrow l_{SI} \approx 0.8\text{ kpc}
\]

\[
\mathcal{L} = \frac{\hbar^2}{2m} g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - \frac{1}{2} mc^2 |\psi|^2 - \frac{\lambda}{2} |\psi|^4,
\]

- Global U(1) symmetry ⇒ Charge (particle number density) conservation

\[
Q \equiv n - \bar{n} = \rho_{SFDM,0} / (mc^2)
\]

- Tensor-to-scalar ratio: \( r \)

- Reheating temperature: \( T_{\text{reheat}} \)
Holistic Evolution of the \( \Lambda \)SFDM Universe

- **Friedmann equation**

\[
H^2(t) \equiv \left( \frac{da/dt}{a} \right)^2 = \begin{cases} 
H_{\text{inf}}^2, & a < a_{\text{inf}}, \\
H_{\text{inf}}^2 \left( \frac{a_{\text{inf}}}{a(t)} \right)^3, & a_{\text{inf}} < a < a_{\text{reheat}}, \\
\frac{8\pi G}{3c^2} \left[ \rho_r(t) + \rho_b(t) + \rho_A(t) + \rho_{\text{SFDM}}(t) + \rho_{\text{GW}}(t) \right], & a > a_{\text{reheat}},
\end{cases}
\]

SGWB contribution to the expansion history *self-consistently* included

- **Klein-Gordon Equation**

\[
\frac{\hbar^2}{2mc^2} \ddot{\psi} + 3 \frac{\hbar^2}{2mc^2} \frac{\dot{a}}{a} \dot{\psi} + \frac{1}{2} mc^2 \psi + \lambda |\psi|^2 \psi = 0,
\]
Cosmological evolution of SFDM in an FLRW Universe

Compare size of SF oscillation freq $\omega$ to Hubble expansion rate $H$

- Fast oscillation regime ("oscillation"):
  \[ \frac{\omega}{H} \gg 1 \]
  disp. relation: $\omega = \omega (V)$,
  e.g.
  \[ \omega = \frac{mc^2}{\hbar} \sqrt{1 + \frac{2\lambda}{mc^2} |\psi|^2} \]
  "easier"

- Slow oscillation regime:
  \[ \frac{\omega}{H} \ll 1 \]
  "harder"

kinetic energy $\equiv 0$: $w = -1$ CC EOS
kinetic energy $\neq 0$: $w = 1$ stiff EOS ("kination")
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  \[ \frac{\omega}{H} \ll 1 \]
  "harder"
  kinetic energy $\equiv 0$: $w = -1$ CC EOS
  kinetic energy $\neq 0$: $w = 1$ stiff EOS ("kination")
  
Behavior is determined by choice of initial conditions!
„spintessence“ SFDM with repulsive SI has 3 phases

**EOS:** \((p/\rho)_{SFDM} = w(t)\)

1. **Early:** \(w = 1\)  
   (stiff)
2. **Intermediate:** \(w = 1/3\)  
   (radiationlike)
3. **Late:** \(w = 0\)  
   (non-relativistic matter)

→ *change of expansion history!*

\[\Omega_{SFDM} \rightarrow 1\] at early times

**Stiff-SFDM-dominated early Universe**

→ *additional \(N_{\text{eff}}\) during (1) and (2)*

→ *amplifies primordial GWs from inflation during (1)*
ΛSFDM+SGWB: the Universe has 6 eras

Inflation $\rightarrow$ Reheating $\rightarrow$ Stiff-SFDM-dom. $\rightarrow$ Radiation-dom. $\rightarrow$ Matter-dom. $\rightarrow$ $\Lambda$-dom.

$(w=1)$ $(w=0)$ $(w=1)$ $(w=1/3)$ $(w=0)$ $(w=1)$

$T_{\text{reheat}} = 10^3 \text{ GeV}$

$\lambda / (mc^2) = 1 \times 10^{-18} \text{ eV}^{-1} \text{cm}^3$

$m = 8 \times 10^{-21} \text{ eV/} c^2$

$r = 0.01$

$\Omega_i$

$\Omega_{\text{SFDM}}$

$\Omega_{\text{Baryons}}$

$\Omega_{\text{Radiation}}$

$\Omega_{\Lambda}$

$\Omega_{\text{GWs}}$

$N_{\text{eff}} = 3.56 \pm 0.23$
Cosmological evolution in an FLRW Universe of real vs. complex SFDM

Magaña, Matos (2012)

\[ m = 10^{-22} \text{ eV} \]

Li, TRD, Shapiro (1310.6061)

\[ N_{\text{eff}} = 3.71^{+0.47}_{-0.45} \]

\[ (m, \lambda)_{\text{fiducial}} = (3 \times 10^{-21} \text{ eV}/c^2, 1.8 \times 10^{-59} \text{ eV cm}^3) \]
Limiting the duration of the stiff phase after reheating and before BBN constrains SFDM parameters via their contribution to $N_{\text{eff}}$

- for given $r$:
  the smaller the DM mass,
  the later must reheating occur

- Matter-radiation equality:

$$1 + z_{\text{eq}} \equiv \frac{1}{a_{\text{eq}}} = \frac{\Omega_b h^2 + \Omega_c h^2}{\Omega_r h^2 + \Omega_{\text{GW}} h^2}$$

- $N_{\text{eff}}$ during BBN:

$$\frac{\Delta N_{\text{eff,BBN}}(a)}{N_{\text{eff,standard}}} = \frac{\Omega_{\text{SFDM}}(a) + \Omega_{\text{GW}}(a)}{\Omega_{\nu}(a)}$$
Cosmological Constraints on the SFDM parameters
\( \Lambda_{\text{SFDM}} + \text{SGWB:} \) enhanced signal of inflationary SGWB due to DM!

Any grav. waves which enter horizon while the background EOS obeys \( w > 1/3 \) amplifies \( \Omega_{\text{GW}} \)!

(Grishshuk, Giovannini, Boyle, …..)

Stiff-SFDM-dominated era amplifies SGWB from (standard) inflation: can be measured/constrained by GW laser interferometers!

\[ \Omega_{\text{GW}}(f) = \Omega_{\text{GW, peak}} \times \begin{cases} f / f_{\text{peak}}, & f \leq f_{\text{peak}} \\ \frac{9 \pi}{64} (f / f_{\text{peak}})^2, & f > f_{\text{peak}} \end{cases} \]

Example 1 prediction for aLIGO/Virgo

![Graph showing the spectrum of \( \Omega_{\text{GW}}(f) \) with various data points and lines representing different observational limits and predictions for different experimental configurations.](image-url)
\textbf{ΛSFDM + SGWB:} enhanced signal of inflationary SGWB due to DM!

Stiff-SFDM-dominated era amplifies SGWB from (standard) inflation: can be measured/constrained by GW laser interferometers!

NEW upper limit from O1 excludes this example case at 95% CL

→ The Age of DM Search/Constraints by GW Detection has begun!
Stiff-SFDM-dominated era amplifies SGWB from (standard) inflation:
can be measured/constrained by GW laser interferometers!

**Example 1 prediction for aLIGO/Virgo**

\[
\frac{\lambda}{mc^2} = 1 \times 10^{-18} \text{ eV}^{-1} \text{cm}^3
\]
\[
m = 8 \times 10^{-20} \text{ eV/c}^2
\]

**ΛSFDM + SGWB:**
Enhanced signal of inflationary SGWB due to DM!
\[ \Lambda \text{SFDM} + \text{SGWB:} \]

enhanced signal of inflationary SGWB due to DM!

Stiff-SFDM-dominated era amplifies SGWB from (standard) inflation:
can be measured/constrained by GW laser interferometers!

Example 2 prediction for aLIGO/Virgo
**ASFDM + SGWB:**

enhanced signal of inflationary SGWB due to DM!

Stiff-SFDM-dominated era amplifies SGWB from (standard) inflation:
can be measured/constrained by GW laser interferometers!

**Example 3 prediction for aLIGO/Virgo**
\[ \Lambda_{SFDM} + SGWB: \]

enhanced signal of inflationary SGWB due to DM!

Stiff-SFDM-dominated era amplifies SGWB from (standard) inflation:
can be measured/constrained by GW laser interferometers → FORECASTS

![Graph](image)

Marginally allowed \( \Lambda_{SFDM} \) models for \( \frac{\lambda}{(mc^2)^2} = 1 \times 10^{-18} \text{ eV}^{-1}\text{cm}^3 \)
\textbf{ASFDM + SGWB:}

enhanced signal of inflationary SGWB due to DM!

Stiff-SFDM-dominated era amplifies SGWB from (standard) inflation:
can be measured/constrained by GW laser interferometers!
Current limits: O1 from LIGO (1612.02029)
Conclusions: SFDM is a good DM candidate

• may resolve small-scale problems of CDM *structure formation*

• rich variety of models allows non-standard *expansion histories* in the early Universe:
  particle parameters are constrained by the CMB, BBN, other PTs, primordial GWs from inflation (SGWB)

• puts into reach the *possible detection of the inflationary SGWB*:
  a wide range of DM particle parameters and reheat temperatures can be already tested by aLIGO/VIRGO O1 run, and more with O5! (some examples are already ruled out from O1)

→ ongoing/upcoming GW laser interferometer experiments can detect/constrain DM!