Classical self-interaction and its effects on motion

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How do things move?

Charged-particle motion

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But test particles are only an idealization.

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Naive point-particle limits fail, both physically and mathematically.

The main difficulty

Self-force

What is the (net) force that something exerts on itself?

Due to Newton's 3rd law, self-forces vanish in non-relativistic systems.

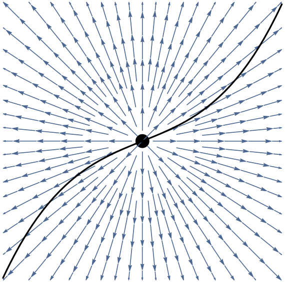
Relativistically,

1. there is no obvious generalization of Newton's 3rd law,
2. fields can carry away energy and momentum.

Intuition for the self-force

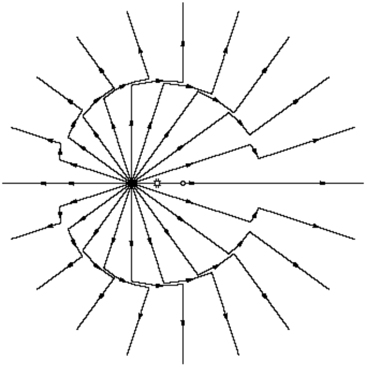
An object carries with it its own field. That field...

1. is highly extended,
2. has its own degrees of freedom,
3. has inertia,
4. can "break away."



Radiation reaction

Objects coupled to long-range fields can radiate.



They must accelerate in reaction to their radiated momentum.

Radiation reaction II

Not only radiation reaction

Momentum carried by radiation implies a (self-) force:

- Balance laws can sometimes be used to calculate said force.
- But objects don't really "care" what's happening to fields far away from them. Try to understand things locally.

Radiation reaction is also incomplete; there are nonradiative self-forces. . .

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Long list of contributors: Thomson, Planck, Lorentz, Poincaré, Dirac, Landau, Lifshitz, Feynman, Wheeler, DeWitt, . . .

An old result

Non-relativistically and in flat 4D spacetime,

$$m\ddot{z}_i = qE_i^{\mathrm{ext}} + \frac{2}{3}q^2 \dddot{z}_i - \mu \ddot{z}_i.$$

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This can be rewritten as

$$\hat{m}\ddot{z}_i = qE_i + \frac{2}{3}q^2 \dddot{z}_i,$$

where

$$\hat{m} := m + \mu. \qquad \text{[Effective mass]}$$

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3. *Runaway solutions*: Even if $E_i^{\mathrm{ext}} = 0$, there are solutions $z(t) = \bar{z}e^{t/\tau}$ with $\tau \sim q^2/m$. For an electron, $\tau \sim 10^{-23}\,\mathrm{s}$!!

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Despite a long period of confusion, these issues are now understood.

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More generally, the self-force involves all of the past history:

$$f_i^{\mathrm{self}}(t) = q^2 \int g_i(z(t), z(t'))dt'.$$

Preacceleration and runaway problems

Artifacts of the approximation and/or invalid attempts to force point particles into the theory.

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No problems for extended charges as long as (self-energy) $\lesssim m$.

Other contexts

What about self-force...

1. with different boundary or initial conditions,
2. in curved spacetime,
3. with higher-order multipole moments,
4. due to gravitation?

Or self-*torque*?

A organizing principle

In every known context, self-force results can be summarized by

Detweiler & Whiting (2002), AIH (2008-2018)

1. Start with test-body laws of motion.
2. Replace all potentials/metrics/... in those laws by $\phi \mapsto \hat{\phi}[\phi]$, for some particular *effective external field* $\hat{\phi}$.

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$\hat{\phi}[\phi]$ is. . .

1. nonlocal,
2. usually linear,
3. usually satisfies the homogeneous field equation.

Example I: Newtonian gravity

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2. Replace all potentials/metrics/... by $\phi \mapsto \hat{\phi}[\phi]$.

   Here, the correct effective field is the external field:

$$\hat{\phi}[\phi; \mathbf{x}] = \phi(\mathbf{x}) - \underbrace{\left(-\frac{1}{4\pi G} \int \frac{\nabla^2 \phi(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}' \right)}_{\text{self-field}}.$$

So,

$$m\ddot{z}_i = -m\nabla_i \hat{\phi}$$

for a *self-gravitating* mass.

Example II: Electromagnetism

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1. Start with test-body laws of motion.

   For a pointlike *test* charge, $m\ddot{z}_a = qF_{ab}\dot{z}^b$.

2. Replace all potentials/metrics/... by $A_a \mapsto \hat{A}_a[A]$.

   Here, the correct external field is

$$\hat{A}_a[A] = A_a(x) - \underbrace{\left(\int G_a{}^{a'}(x,x')J_{a'}[A](x')dV' \right)}_{\text{self field}}$$

for a particular *Green function* $G_a{}^{a'}$.

Example II: Electromagnetism (cont.)

Given $\hat{A}_a[A]$ and $\hat{F}_{ab}[A] = 2\nabla_{[a}(\hat{A}_{b]}[A])$,

$$\hat{m}\ddot{z}_a = q\hat{F}_{ab}\dot{z}^b$$

for a *self-interacting* charge.

1. $\hat{F}_{ab}$ satisfies the source-free Maxwell eqns.
2. Mass is renormalized: $m \mapsto \hat{m}$. Charge isn't.

Effective EM field

For a small charge in flat spacetime,

$$\hat{F}_{ab} = F_{ab}^{\mathrm{ext}} + \frac{4}{3} q \dot{z}_{[a} \ddot{z}_{b]} + \dots$$

1. Very different from F_{ab}.
2. Non-singular even for a point particle.
3. Reproduces old results.

Gravitational self-force

Test bodies fall on geodesics: $\dot{z}^b \nabla_b \dot{z}^a = 0$.

Self-gravitating bodies also fall on geodesics, but in an effective metric $\hat{g}_{ab} = \hat{g}_{ab}[g]$:

$$\dot{z}^b \hat{\nabla}_b \dot{z}^a = 0$$

$\hat{g}_{ab}$ is well-defined but difficult to compute. Depends on all past history.

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Example: Everything couples to the metric $\Rightarrow$ all multipole moments of T_{ab} are generically renormalized [Linear momentum (incl. mass), angular momentum, mass and momentum quadrupoles, . . .]

Conclusions

1. Self-force is one aspect of the problem of motion.
2. Foundations largely understood and unified.
3. Self-interaction renormalizes test-body parameters and generalizes test-body laws of motion with effective fields.
4. Explicit calculations can still be difficult.