Cosmic Strings

some new results

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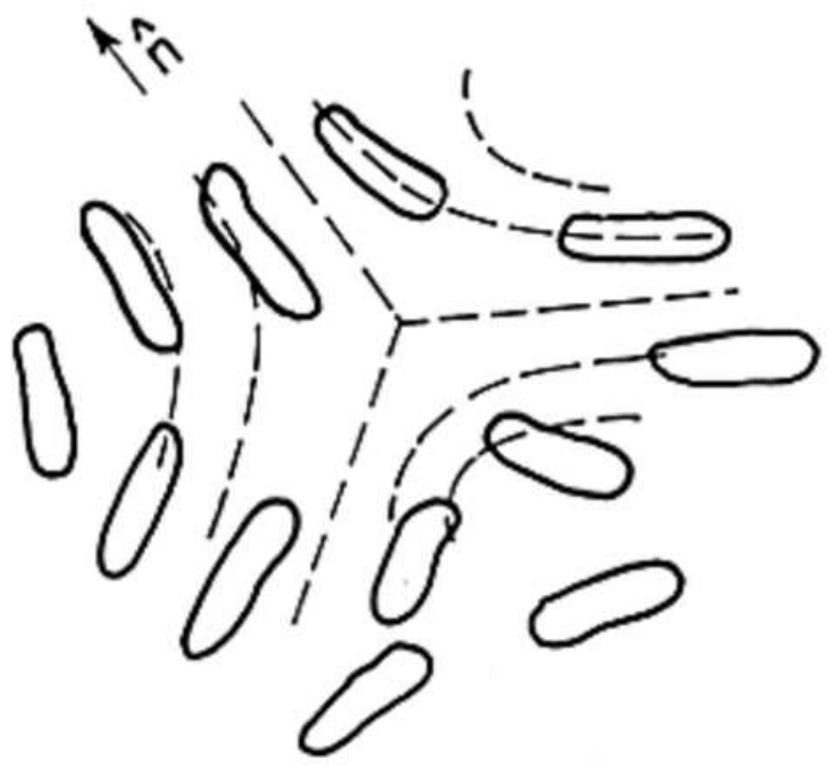
Plan

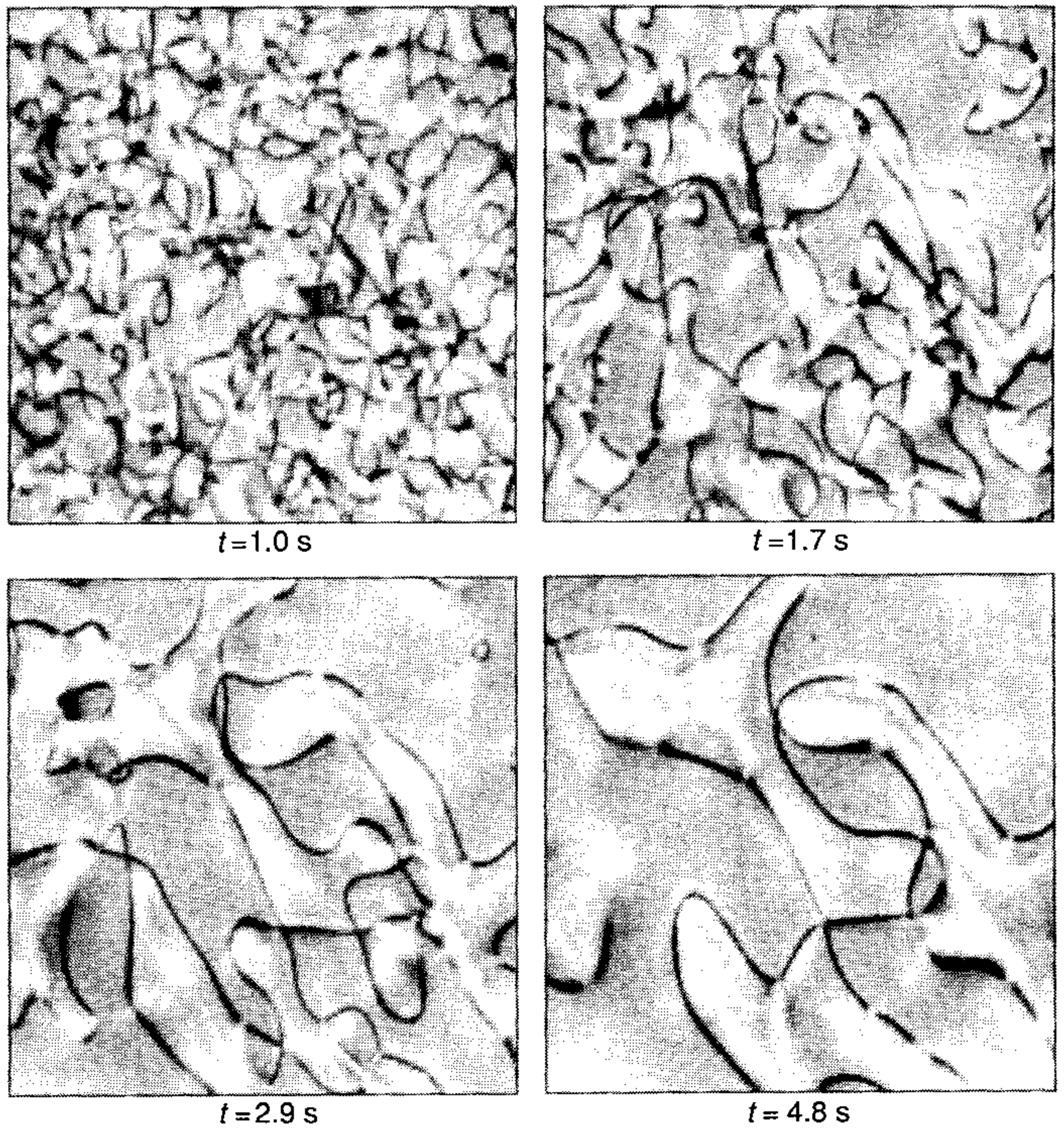
3 projects

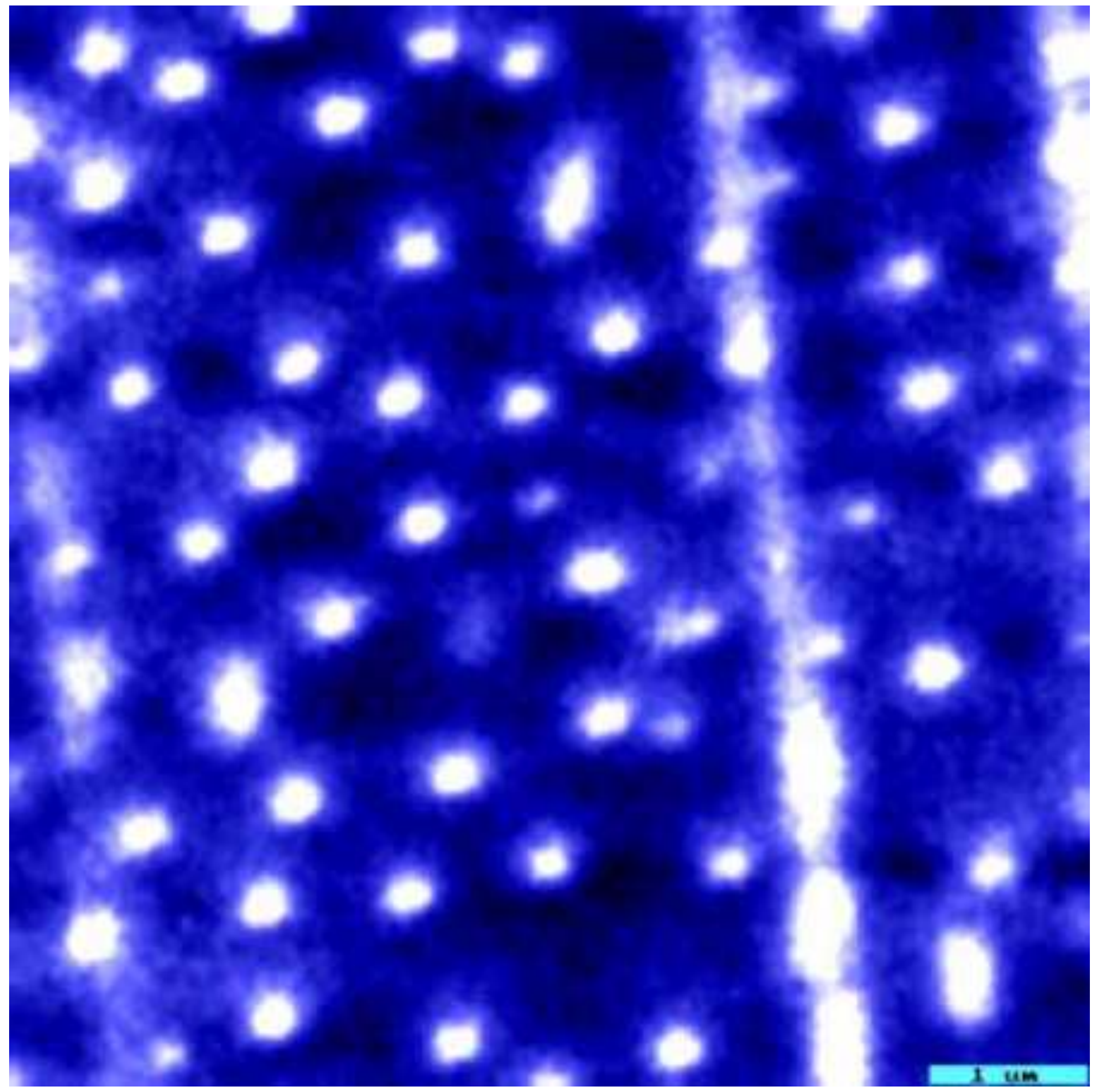
- Quantum formation of topological defects.
- Evolution of global string loops.
- Evolution of gauge string loops.

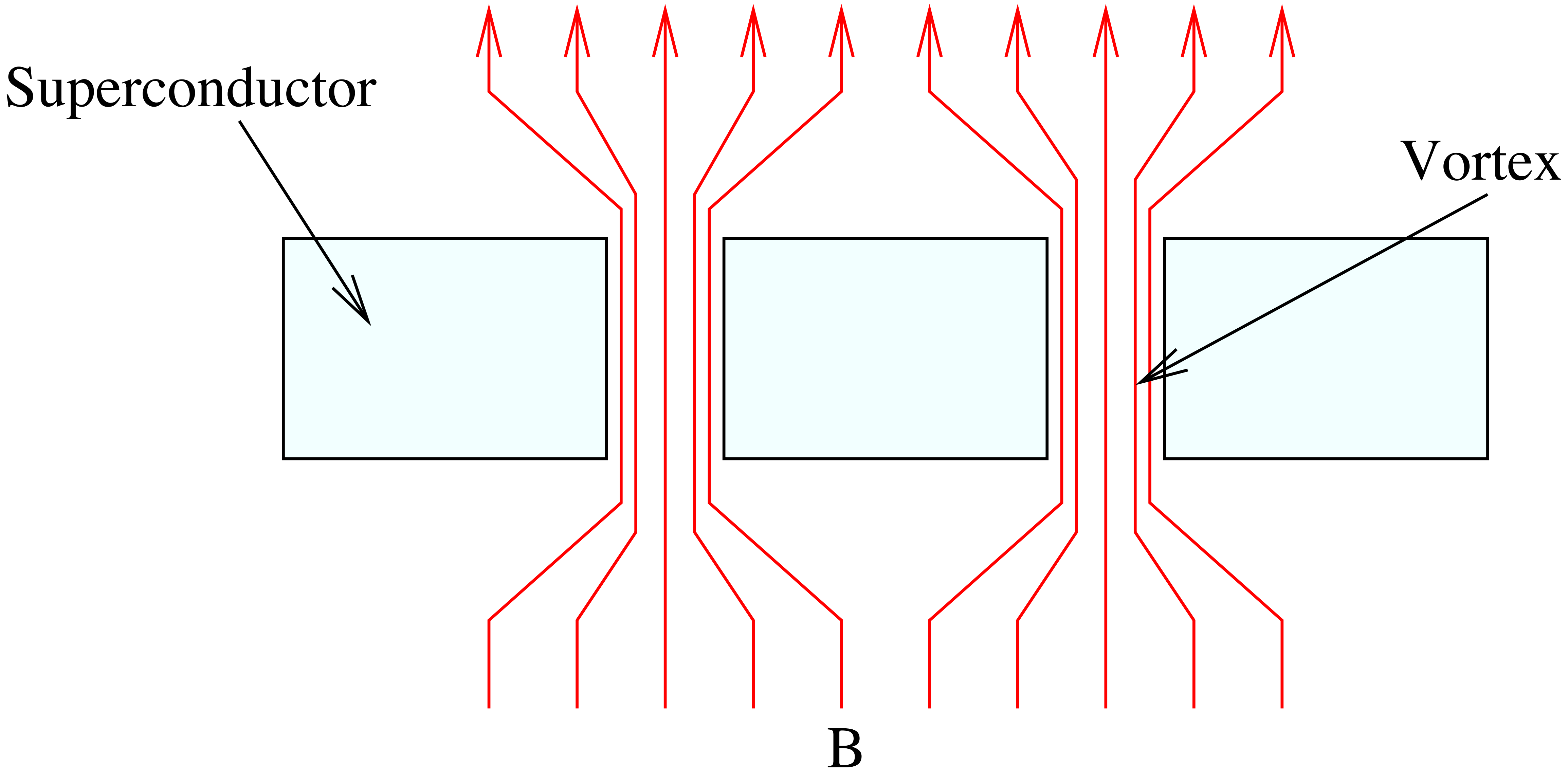
Examples of topological defects

Nematic liquid crystals and superconductors









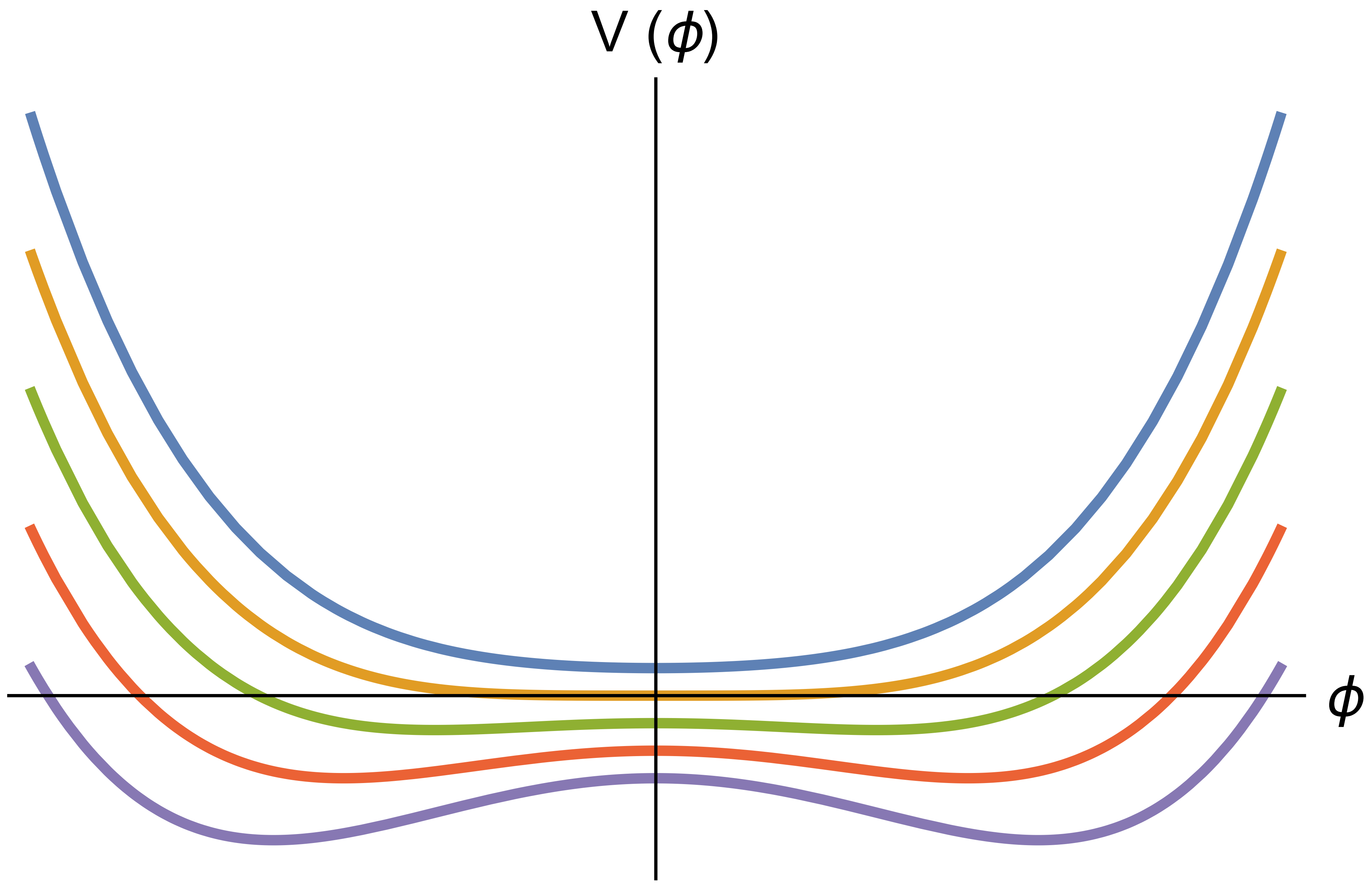
Emergence of classical structures from the quantum vacuum

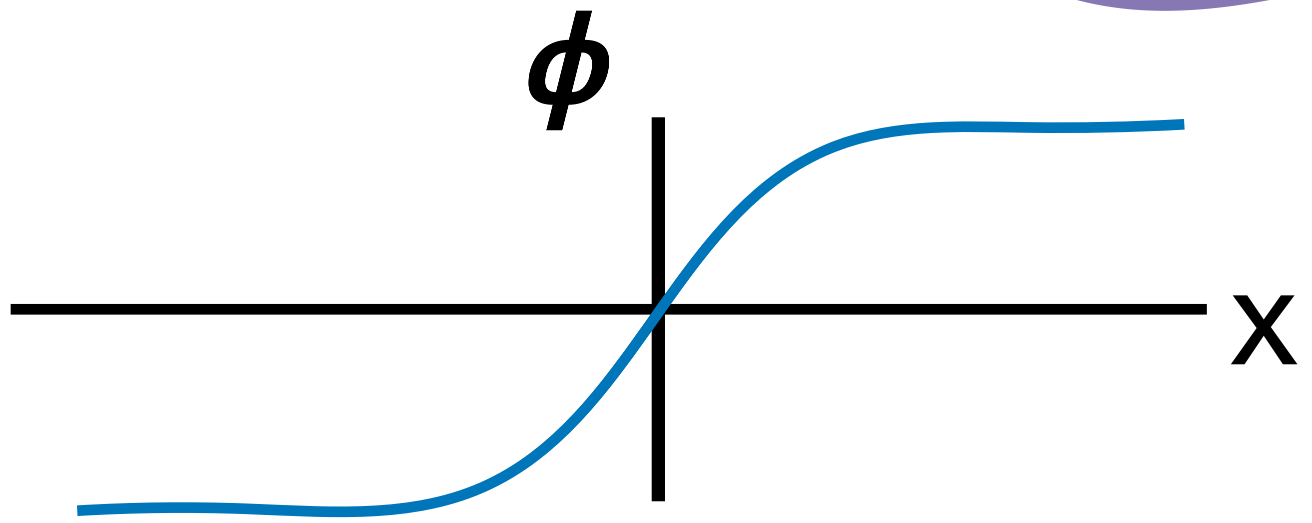
A full quantum calculation (kinks first)

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_2(t)\phi^2 - \frac{\lambda}{4}\phi^4$$

$$m_2(t) = -m^2 \tanh\left(\frac{t}{\tau}\right)$$

$$\phi_K(x) = \frac{m}{\sqrt{\lambda}} \tanh\left(\frac{mx}{\sqrt{2}}\right)$$

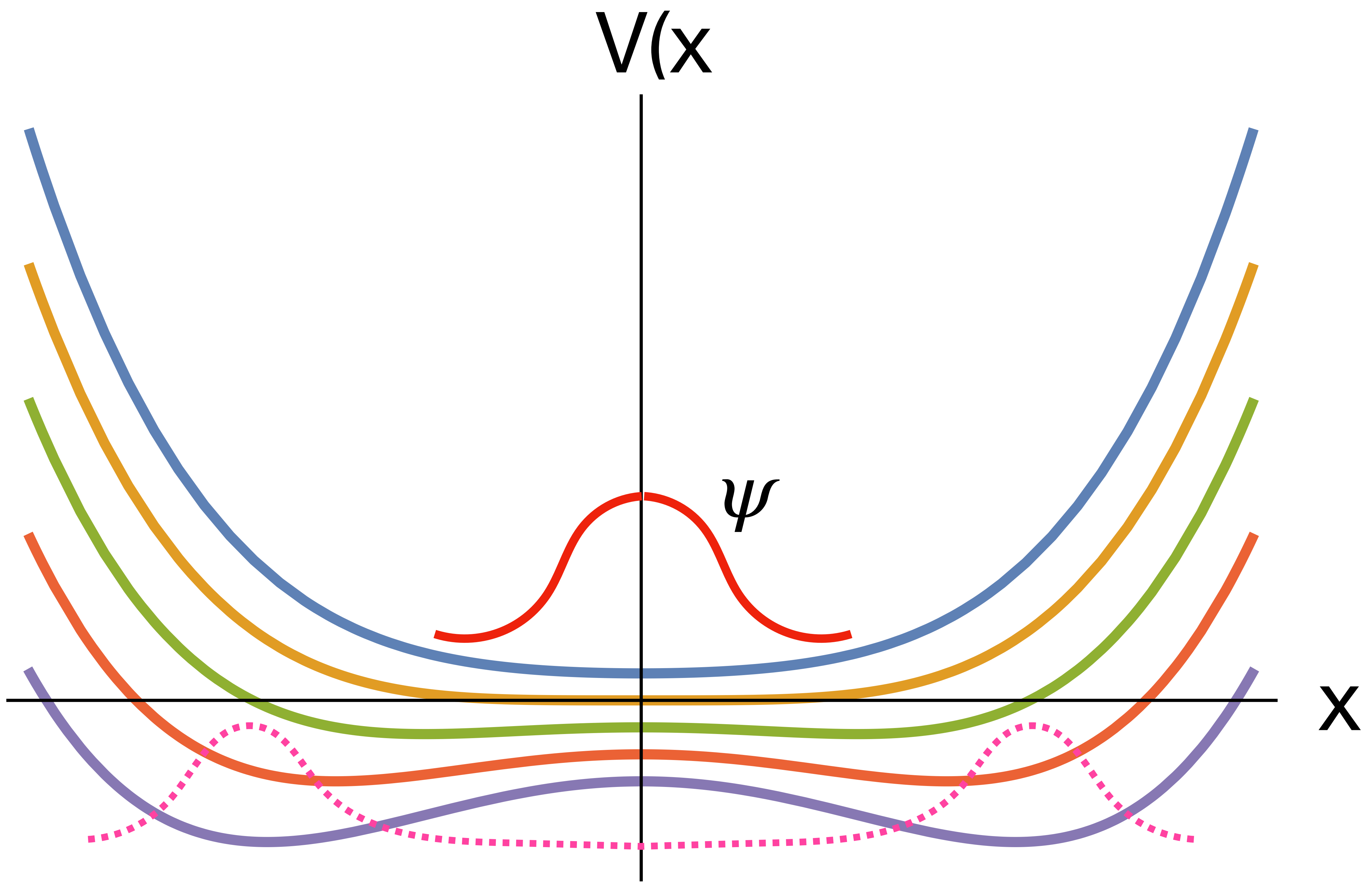




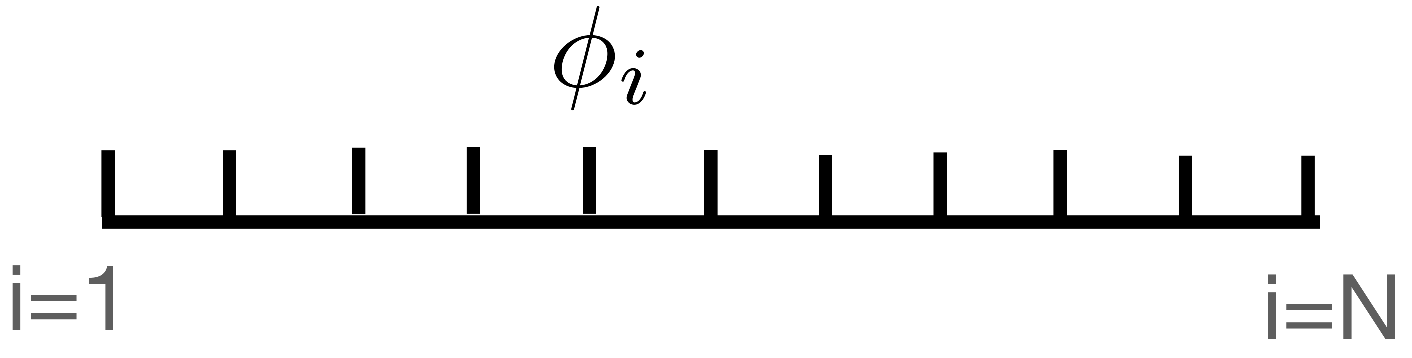
Mainak Mukhopadhyay, George Zahariade & TV, 2004.07249, 2009.11480

A quantum mechanics problem

A toy



but now in quantum field theory:

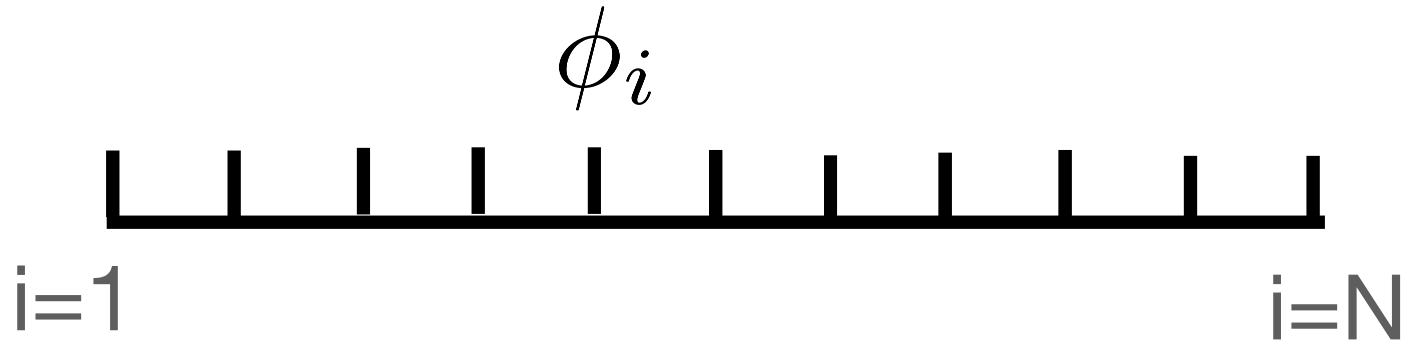


periodic boundaries

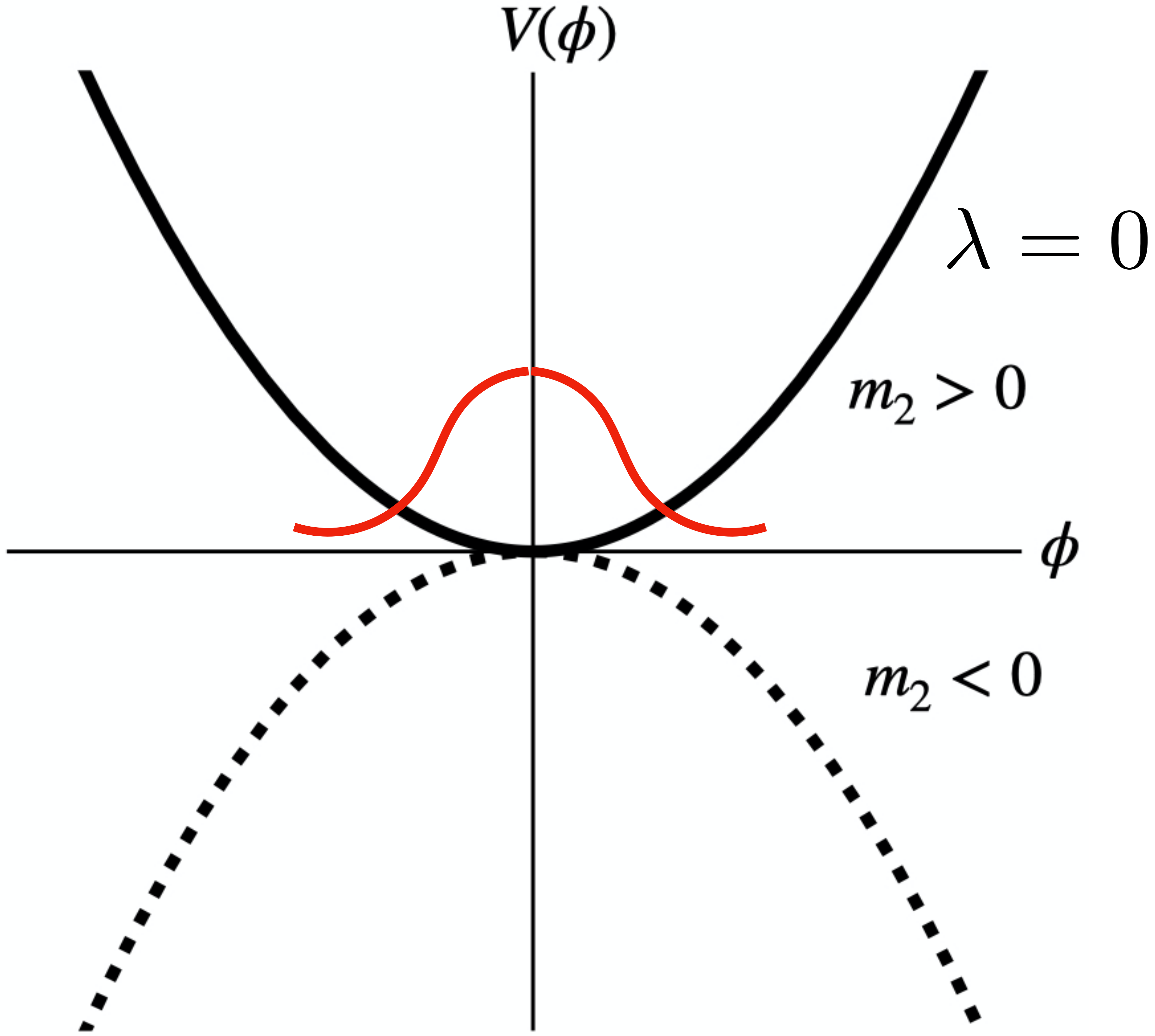
kinks ~ configurations trapped on top of V

Formation

the relevant physics



periodic boundaries



Solve the functional Schrodinger equation

(for the ground state):

$$P[\{\phi_i\}, t] = \Psi^\dagger \Psi = \frac{1}{\sqrt{\det(2\pi K)}} e^{-\phi^T K^{-1} \phi/2}$$

K is a time dependent NxN matrix.

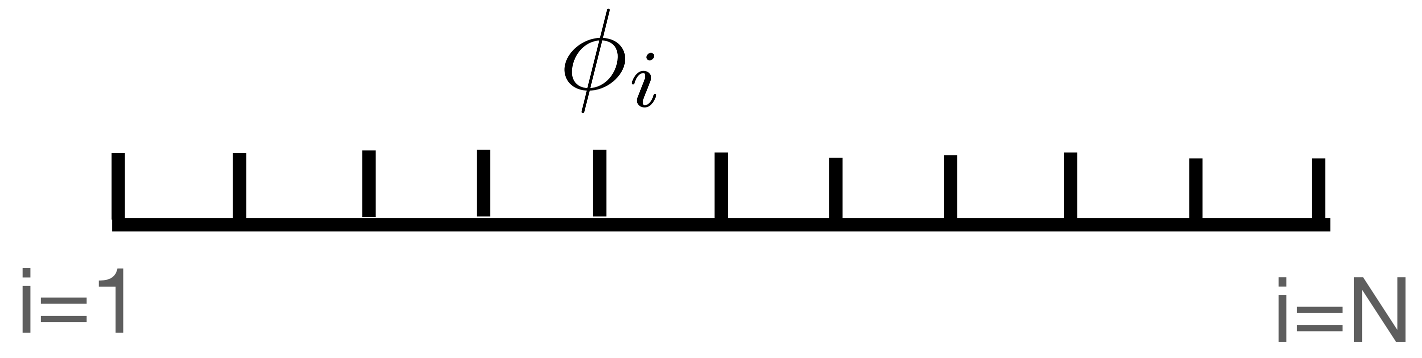
$$K = ZZ^\dagger \qquad \ddot{Z} + \Omega_2(t) Z = 0 \qquad \text{(!)}$$

Z is a time dependent, complex NxN matrix.

$$\Omega_2(t) = -\nabla^2 + m_2(t)$$

Formation

counting kinks



Count zeros, *i.e.* sign changes.

$$n_Z = \langle \hat{n}_Z \rangle = \frac{N}{2L}\left[1 - \left\langle \mathrm{sgn}\left(\hat{\phi}_1 \hat{\phi}_2\right)\right\rangle\right]$$

Use translational invariance.

$$\left\langle \mathrm{sgn}\left(\hat{\phi}_1 \hat{\phi}_2\right)\right\rangle = \frac{1}{\sqrt{\det(2\pi K)}} \sum_{\text{quads.}} \int d\phi_1 \ldots d\phi_N \, \mathrm{sgn}(\phi_1 \phi_2)\, e^{-\phi^T K^{-1} \phi/2}$$

$$n_K = \frac{N}{2L}\left[1 - \frac{2}{\pi}\sin^{-1}\left(\frac{\sum_{|n|\leq n_c} |c_n|^2 \cos(2\pi n/N)}{\sum_{|n|\leq n_c} |c_n|^2}\right)\right]$$

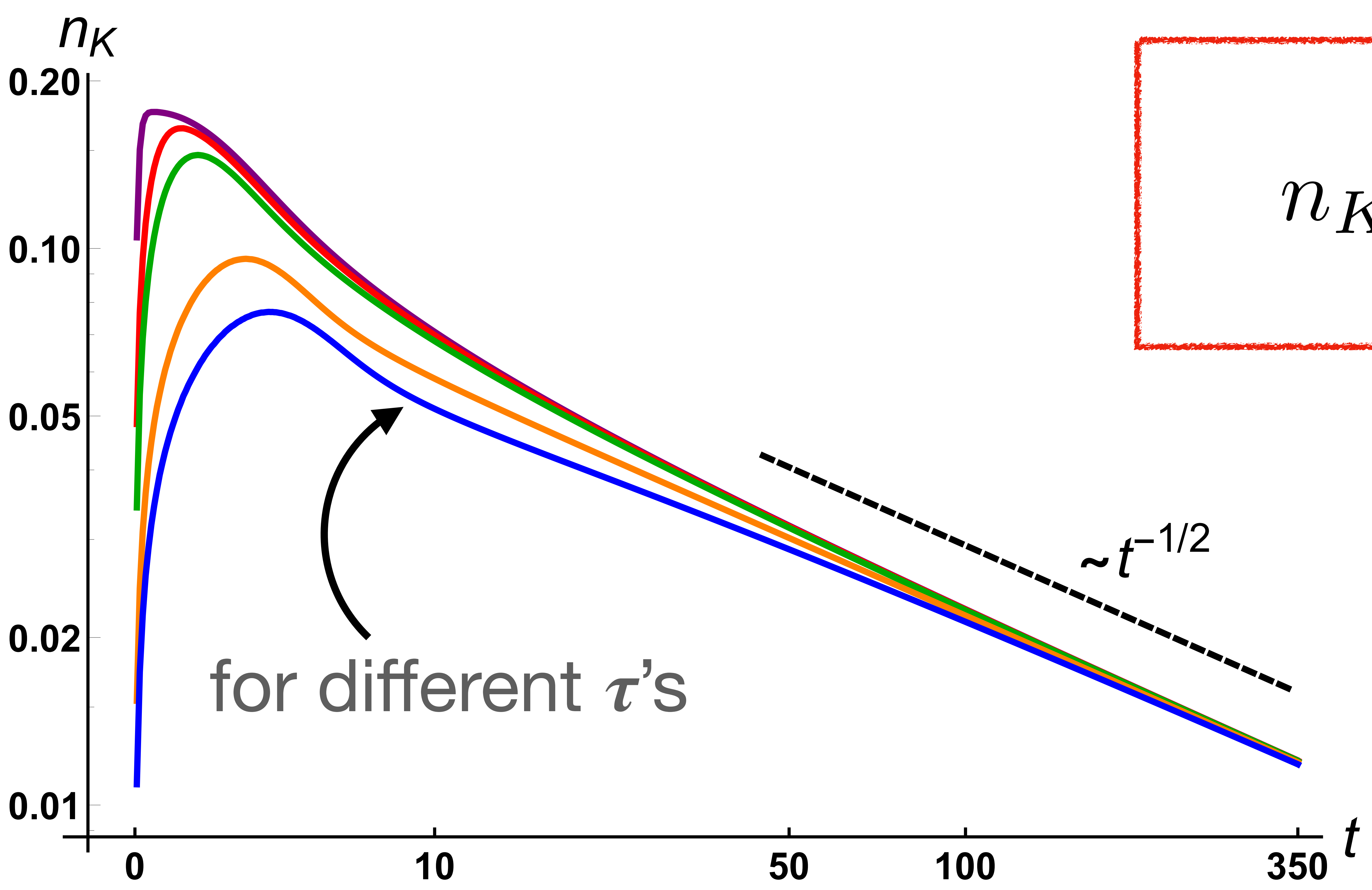
skipping quite a bit of math….

$$\ddot{c}_n + [k_n^2 + m_2(t)]c_n = 0$$

with specified initial conditions.

Formation

results



$$n_K(t) = \frac{1}{\pi}\sqrt{\frac{m}{2t}} + \mathcal{O}(t^{-3/2})$$

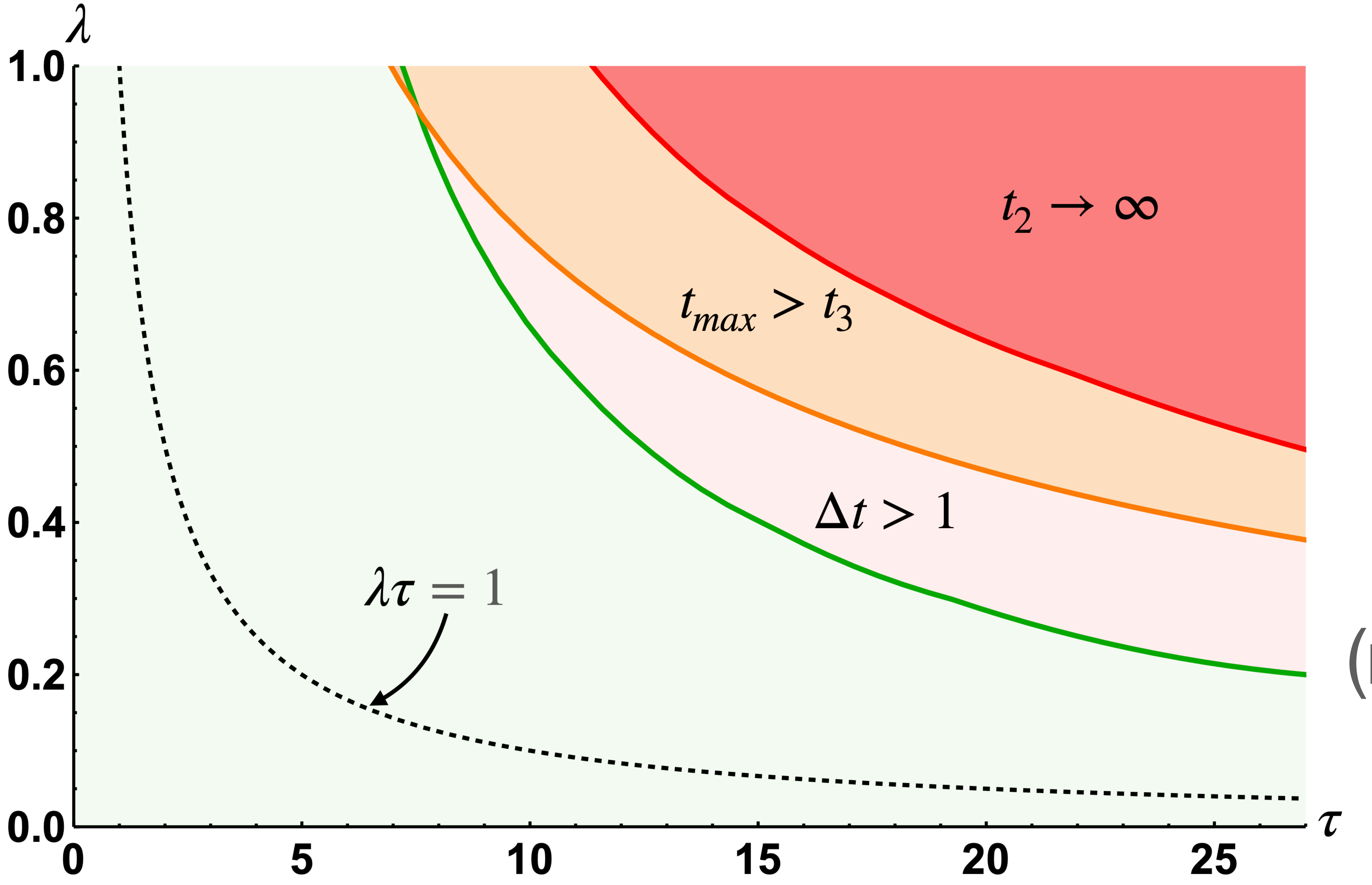
Independent of τ!

Compare: Kibble-Zurek

Applicability

non-zero λ

Perturbation parameter: $\sim \lambda\tau/m$



(m=1 units)

Mainak Mukhopadhyay, George Zahariade & TV, arXiv:2009.11480

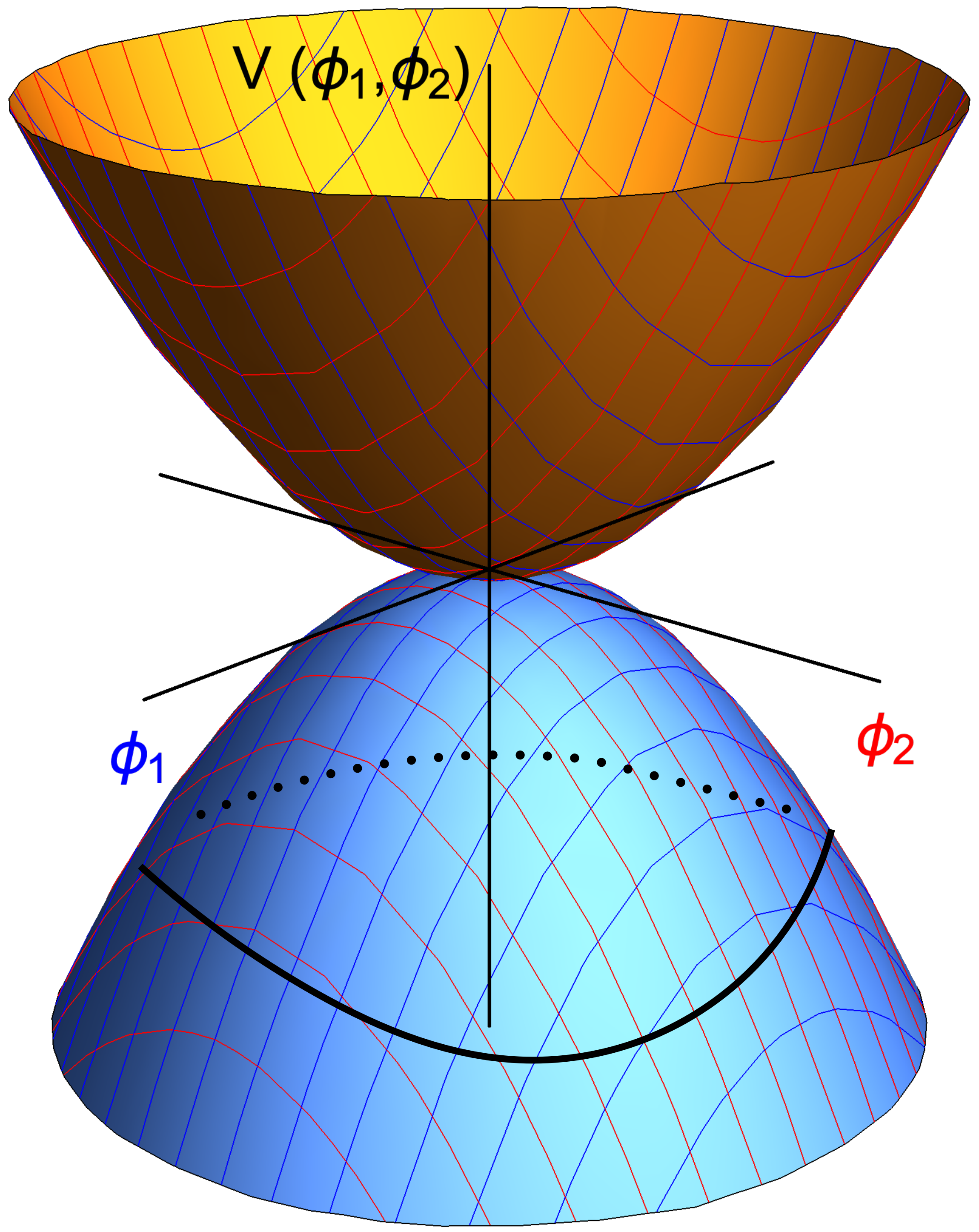
Formation

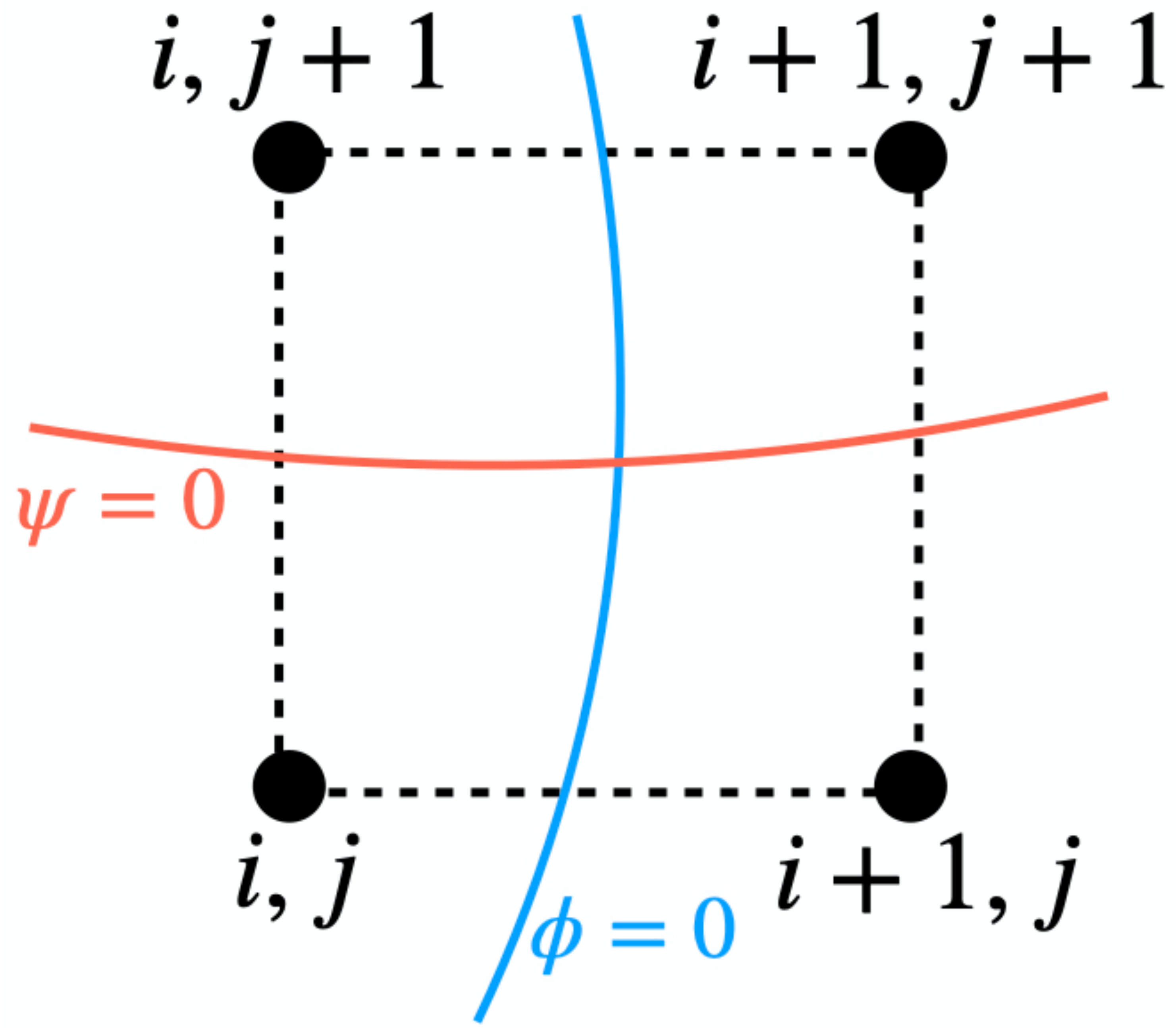
vortices (2 spatial dimensions)

$$\Phi = \phi + i\psi$$

$$L = \frac{1}{2}|\partial_\mu \Phi|^2 - \frac{1}{2}m_2(t)|\Phi|^2 - \frac{\lambda}{4}|\Phi|^4$$

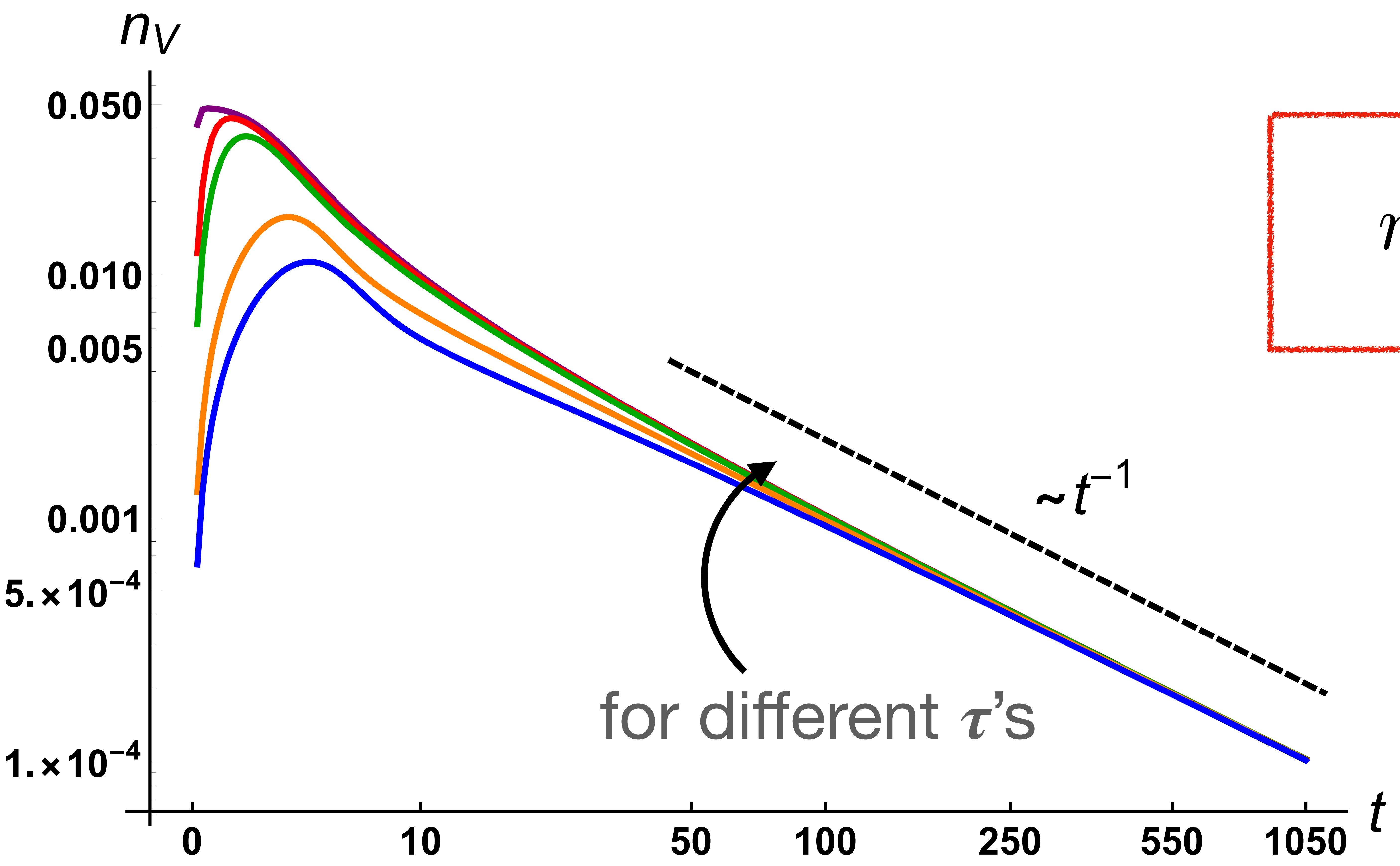
$$m_2(t) = -m^2 \tanh\left(\frac{t}{\tau}\right)$$





Formation

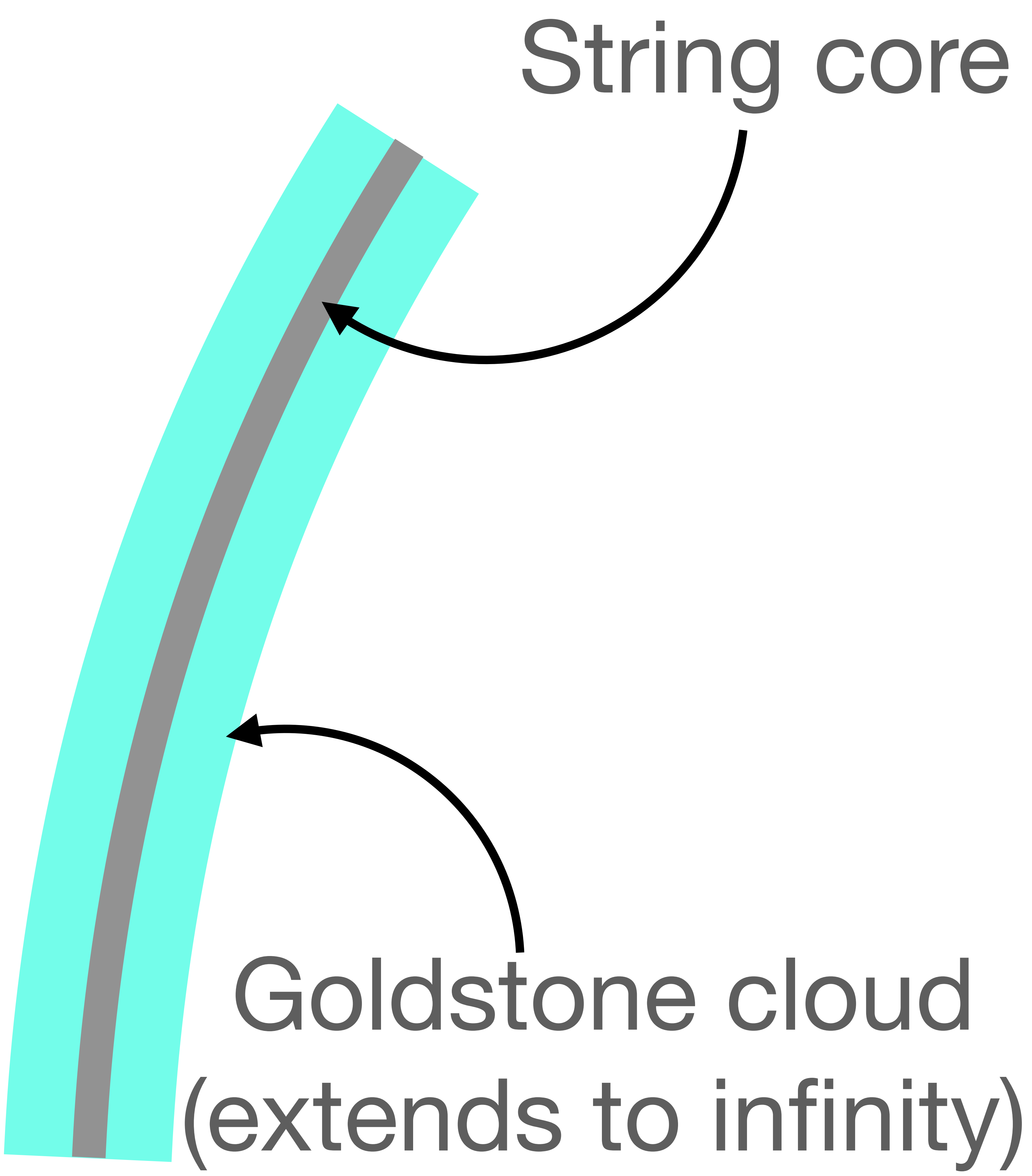
vortices/strings



$$n_V = \frac{m}{\pi^2 t} + \mathcal{O}(t^{-2})$$

Evolution

Global strings (3 dimensions)



$$L = \frac{1}{2}|\partial_\mu \phi|^2 + \frac{1}{2}m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

Relevant to axion models before the QCD phase transition, where $\boldsymbol{\phi}$ is the Peccei-Quinn field. The phase of $\boldsymbol{\phi}$ is the axion field.

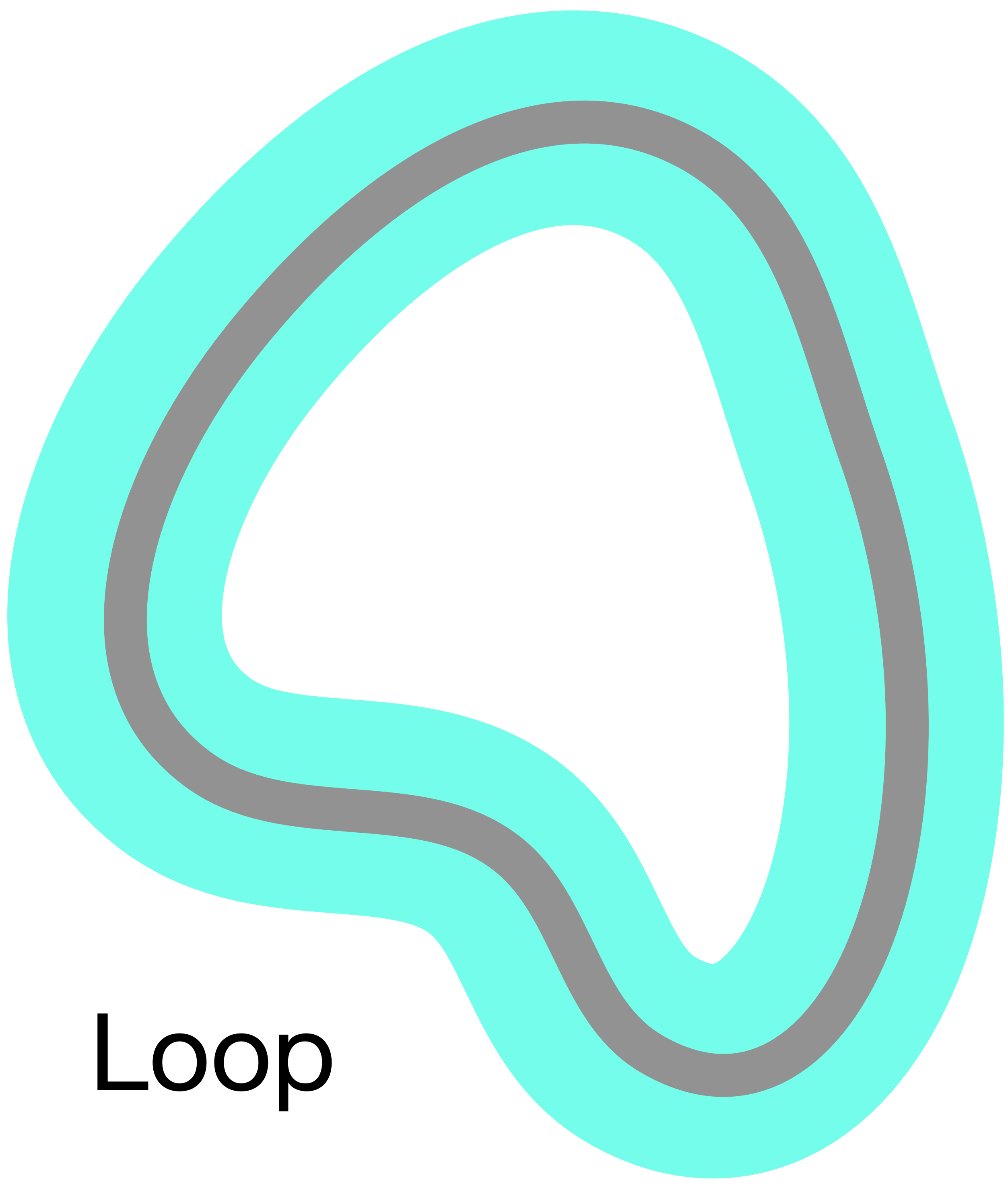
Straight string: $\phi = \eta f(r) e^{i\varphi}$

$$f(r) \sim \text{“}\tanh(r/w)\text{”}$$

Energy density falls of as 1/r because of Goldstone cloud. Similar to electric line charge.

Evolution

Kalb-Ramond dynamics



Kalb-Ramond action in terms of 2-form field:

$$S = -\mu \int d^2\sigma \sqrt{-g_2} + \kappa \int d\sigma^{\mu\nu} A_{\mu\nu} - \frac{1}{6} \int d^4x \, H_{\mu\nu\lambda} H^{\mu\nu\lambda}$$

Nambu-Goto Goldstone cloud

Caveats: No massive radiation. Small backreaction.

Results: Goldstone boson radiation at primary frequency with k ~ 1/L. Loop decays after ~10 oscillations. Tight constraints on QCD axion mass.

A. Vilenkin & TV, 1987; R.L. Davis, 1986; R.L. Davis & E.P.S. Shellard, 1989; …

Evolution

Field theory dynamics

$$\partial_t^2 \phi_a = \nabla^2 \phi_a - \frac{1}{2}(\phi_b \phi_b - 1)\phi_a \qquad a, b = 1, 2$$

Caveats: Initial conditions? Limited by simulation size.

Results: Goldstone boson radiation with 1/k power spectrum.

Loops decay within ~1 oscillation.

More relaxed constraints on QCD axion mass.

C. Hagmann & P. Sikivie, 1991; T. Hiramatsu et al, 2011;
M. Gorghetto, E Hardy & G. Villadoro, 2018; V.B. Klaer & G. Moore, 2019

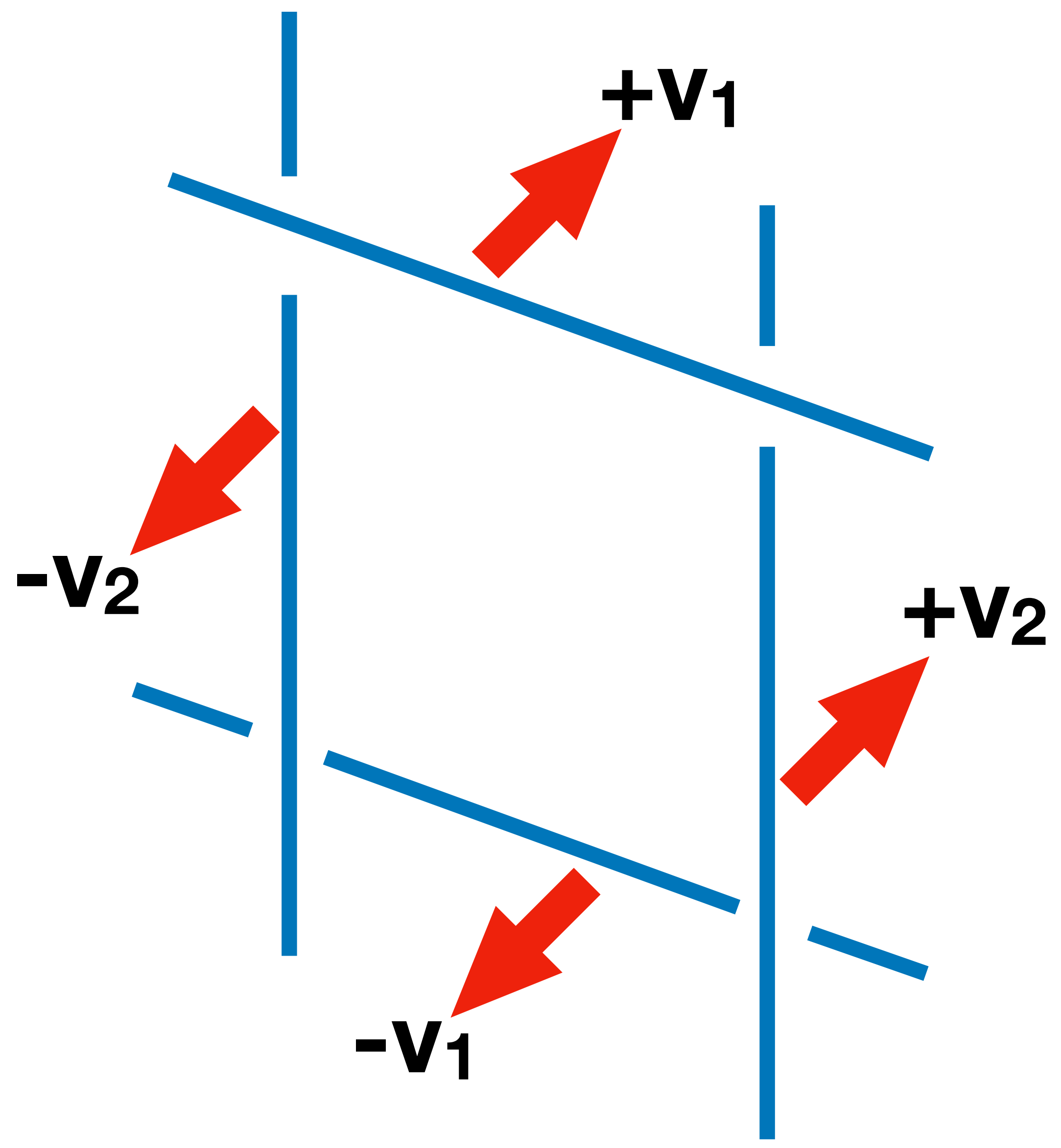
Evolution

Field theory simulations

Parallel on XSEDE

What's a good way to set up the initial conditions?

Use straight string solution and mimic cosmological production of loops.



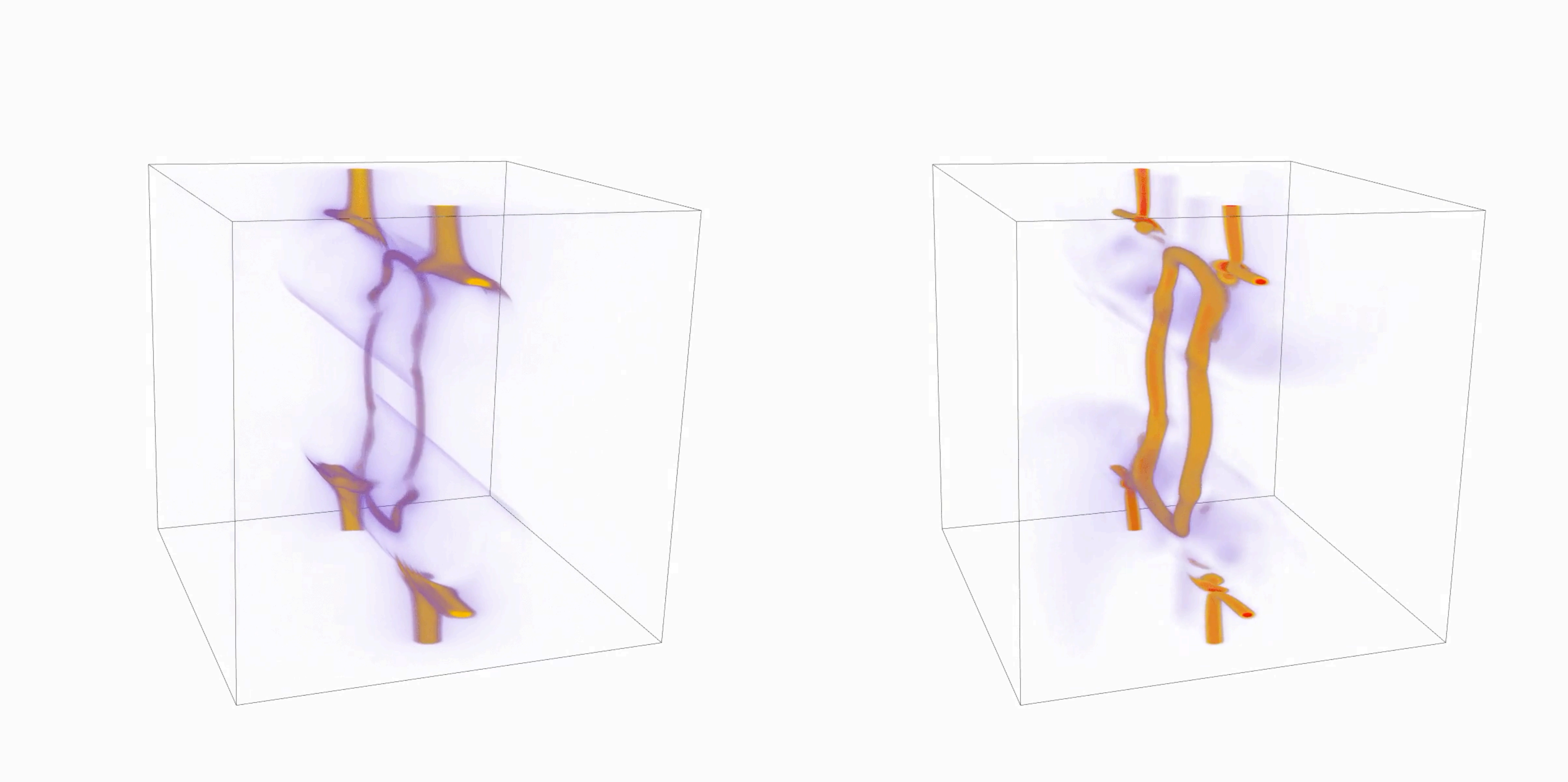
Technical note: Requires Lorentz boosting the static straight string solutions, patching together the four string solutions, and enforcing periodic boundary conditions. The latter requires modifications to the "product ansatz" for patching strings.

A. Saurabh, TV, & L. Pogosian, 2020

Animation

Total energy; potential energy

$|\mathbf{v}| = 0.6$

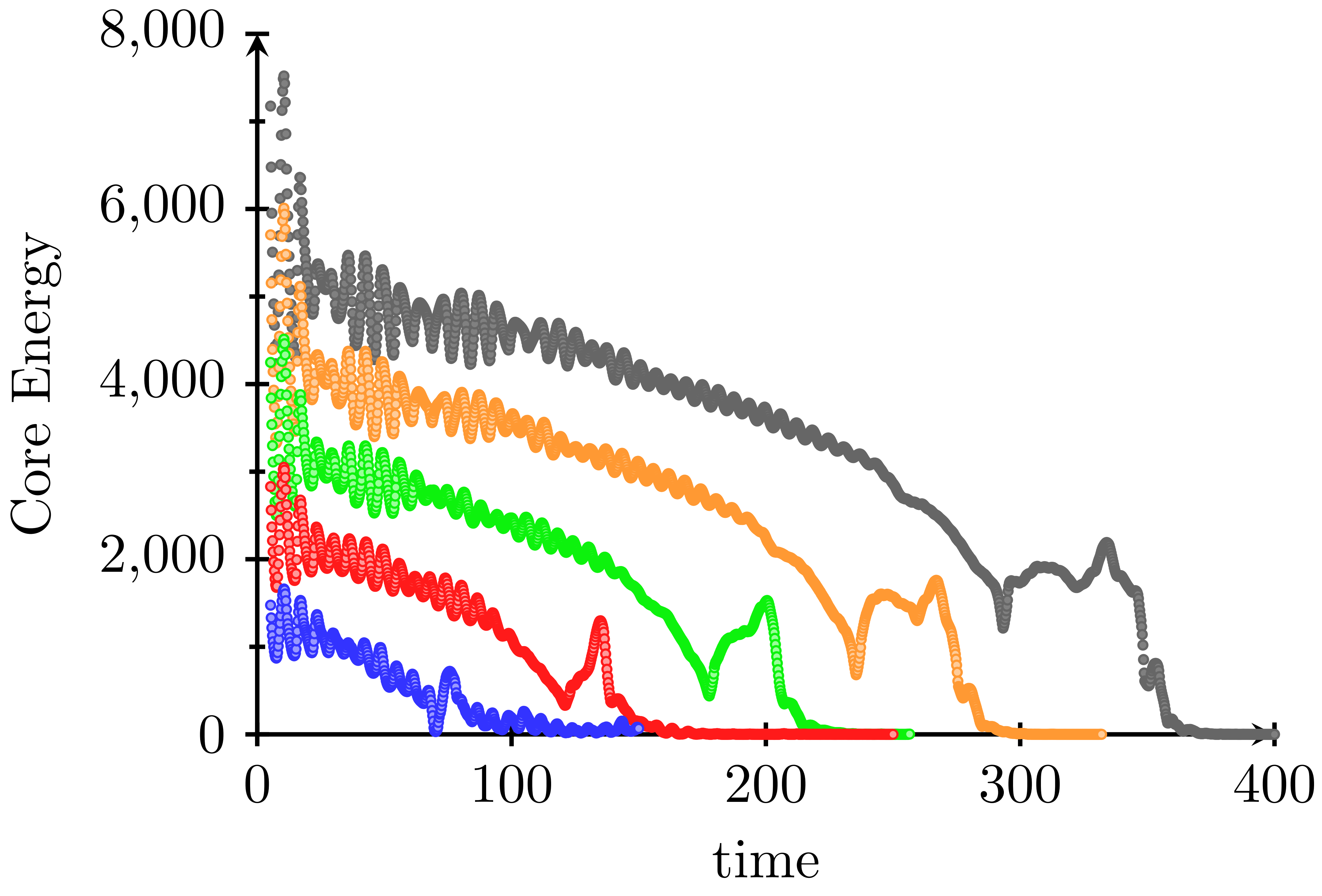


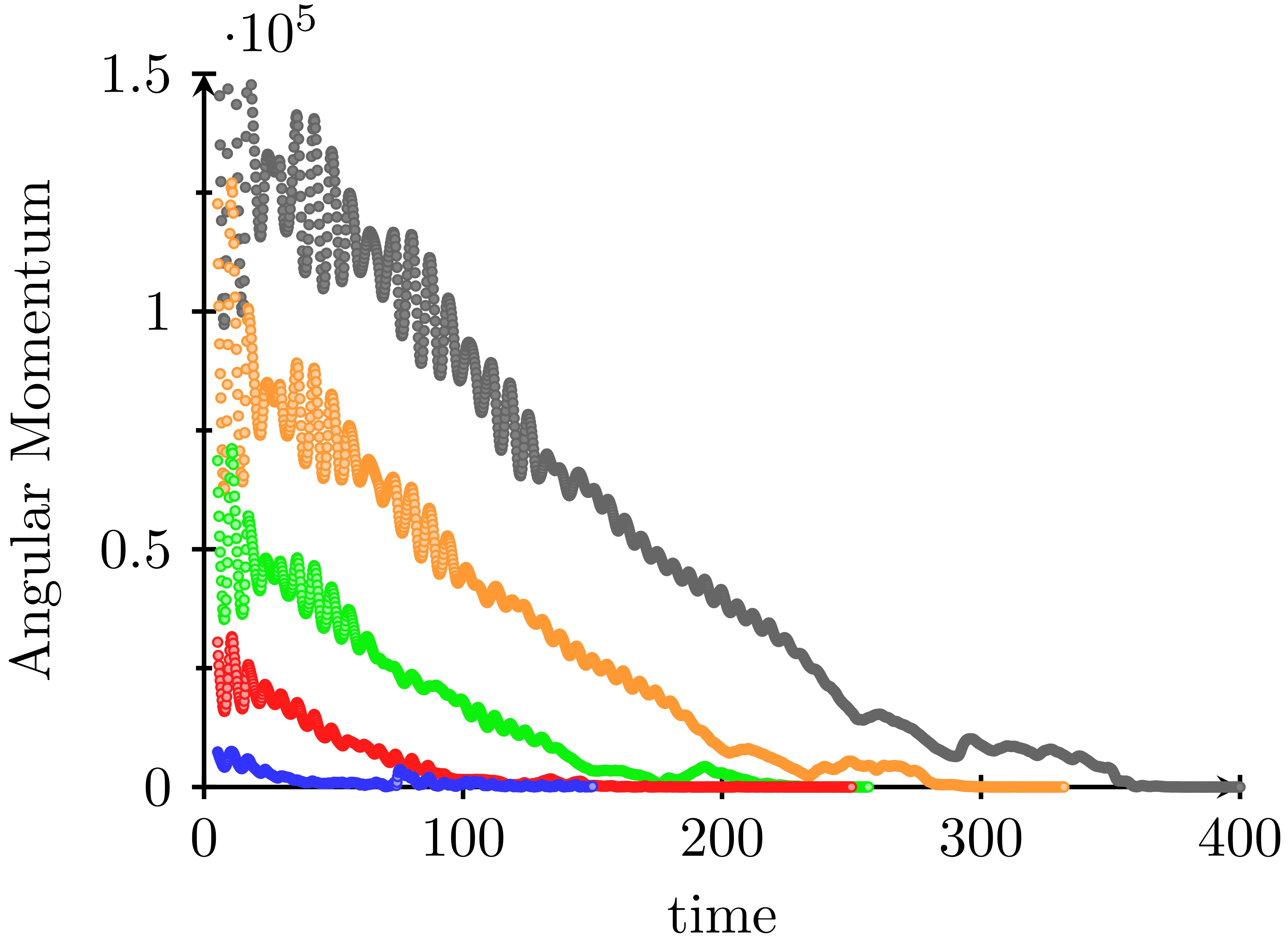
Results

Core energy; angular momentum

Core: $|\phi| < 0.9\eta$

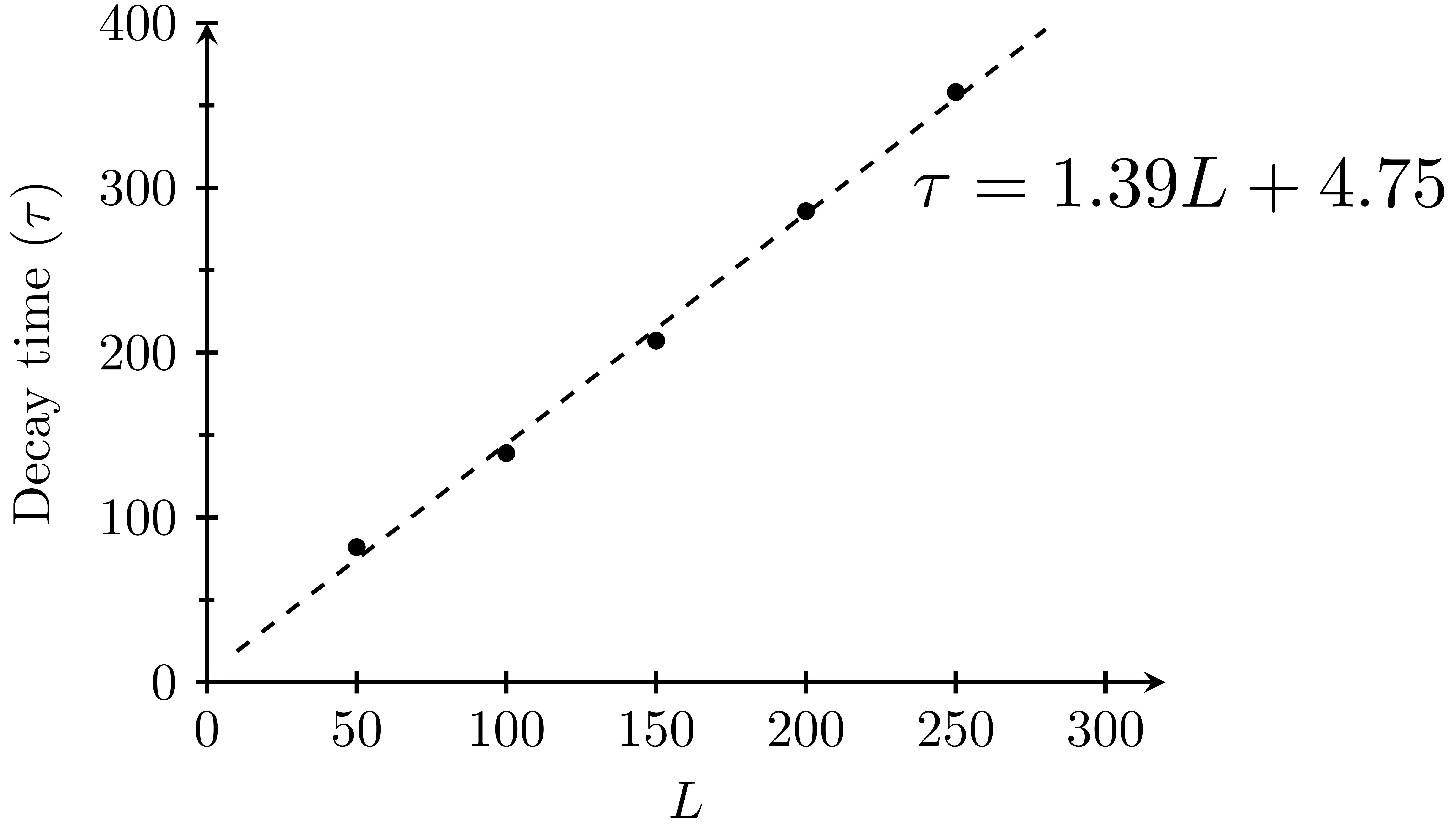
$$L/w = (50, 100, 150, 200, 250) \times 4$$





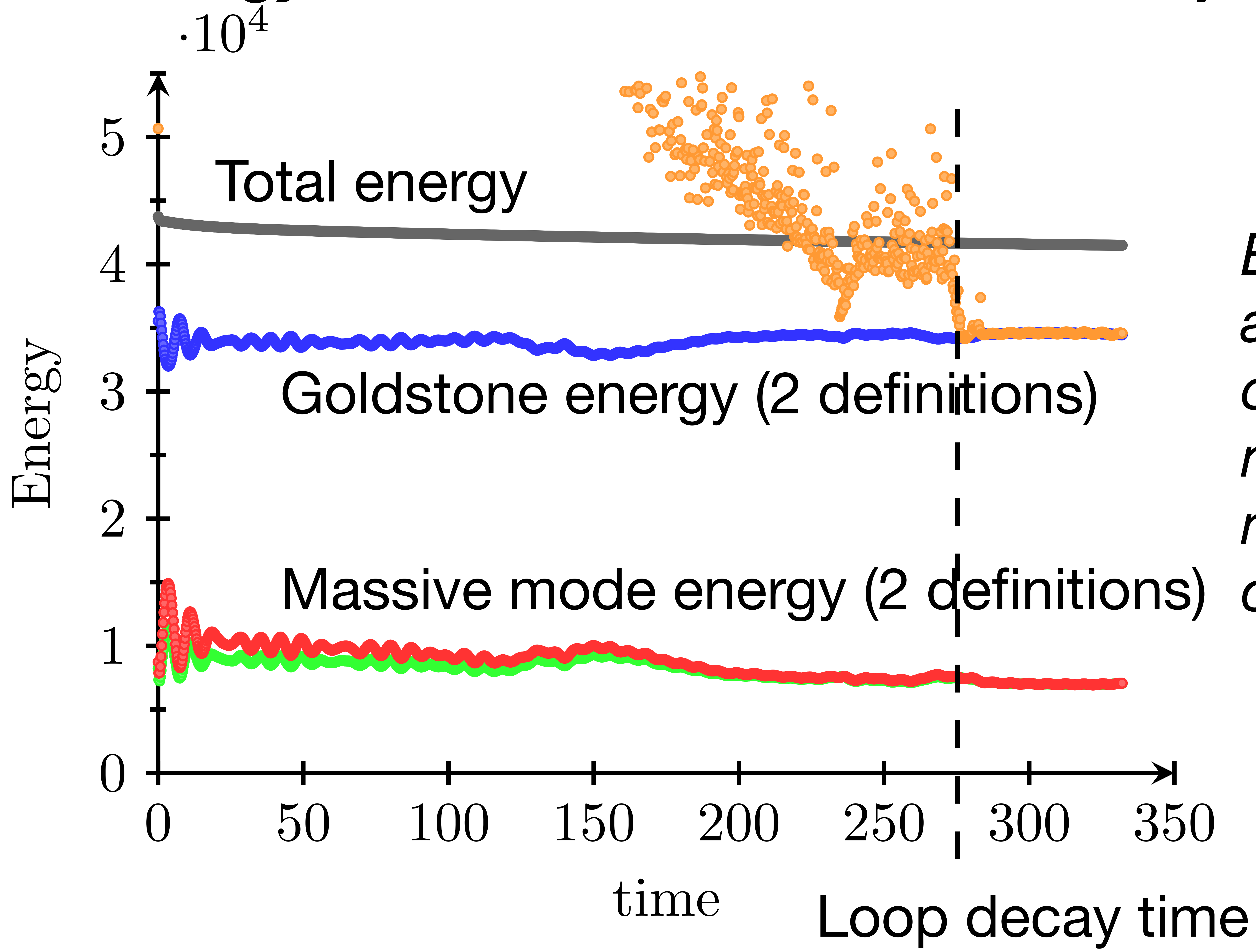
Results

Loop lifetime



Results

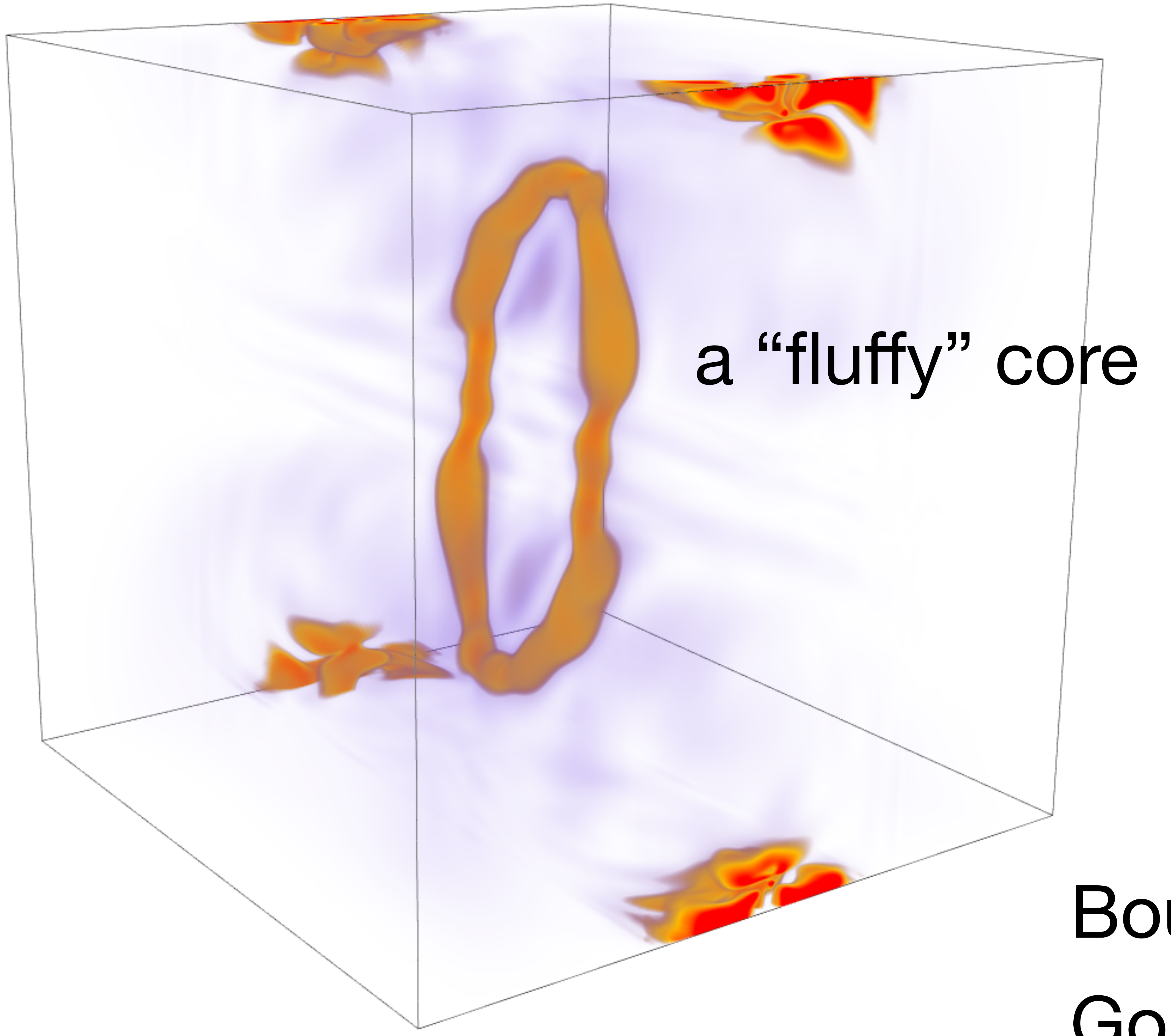
Energy in massive and massless components



Energies in individual components are quite well conserved up to loop decay time: cloud radiates into massless modes, core radiates into massive modes that only later convert into massless modes.

Results

Can we see the creation of massive modes?



Look for bound states in core.

$$\phi = (f(r) + e^{-i\omega t} g(r)) e^{i\theta}$$

$$-f'' - \frac{f'}{r} + \left[\frac{1}{r^2} - \frac{1}{2}(1 - f^2) \right] f = 0$$

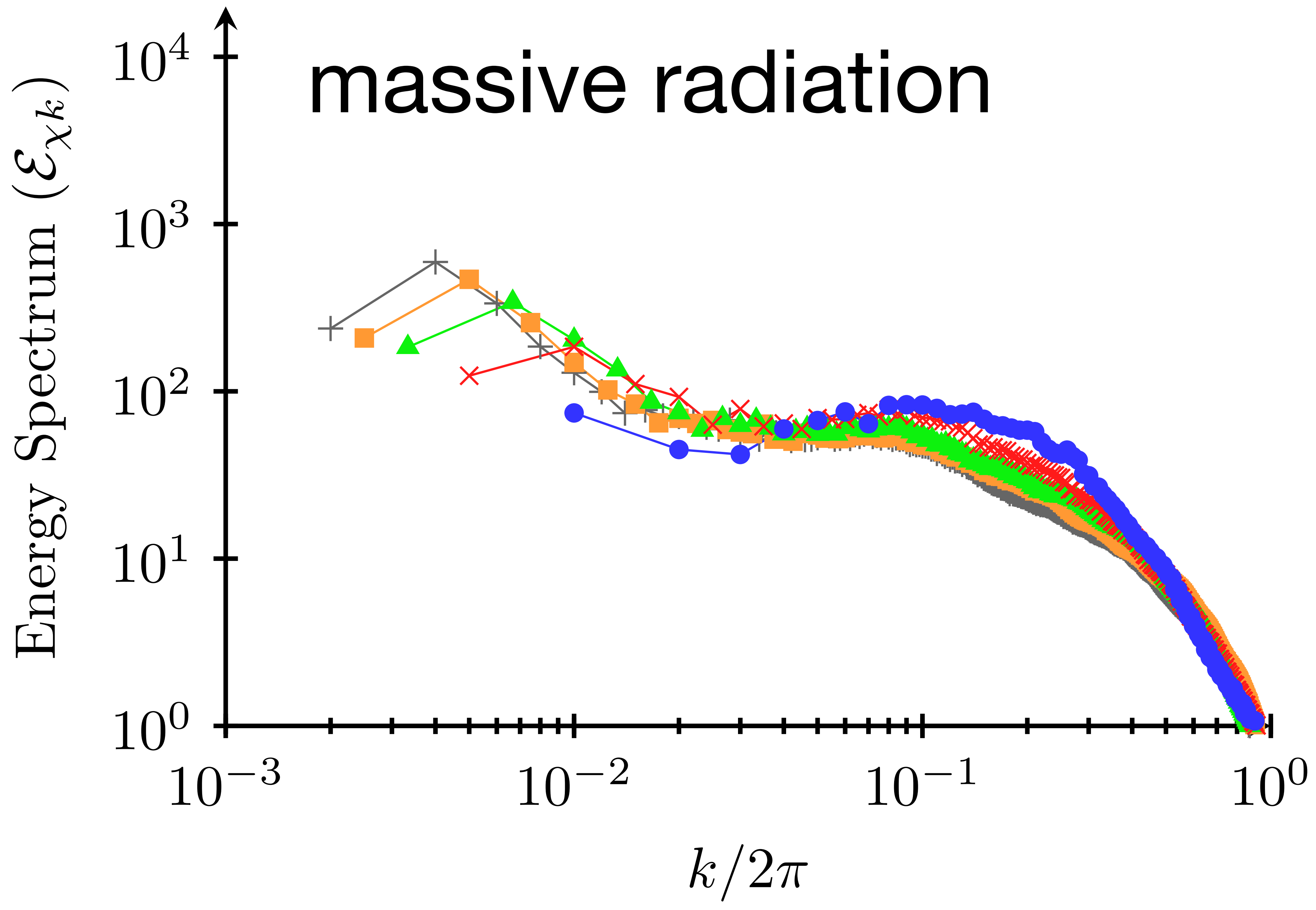
$$-g'' - \frac{g'}{r} + \left[\frac{1}{r^2} - \frac{3}{2}(1 - f^2) \right] g = \Omega g$$

$\Omega \equiv \omega^2 - 1 = -0.19$ implies bound state.

Bound states excited by string intersections and Goldstone boson back reaction (1/r^2 term).

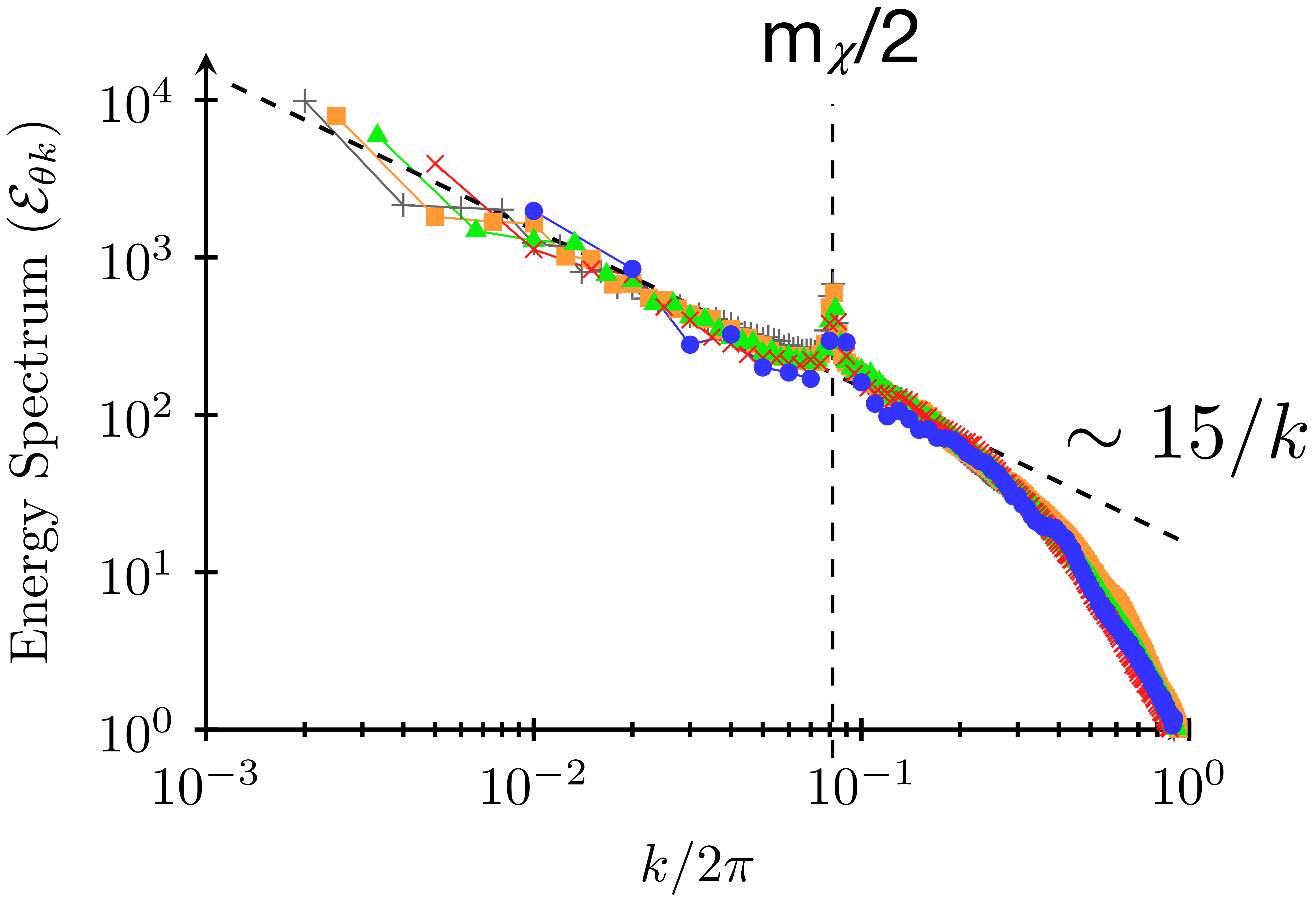
Energy spectra

Massive and massless modes



N=50 (blue), 100, 150, 200, 250.

Goldstone radiation



Summary

Global string loop results and caveats

We have simulated (cosmological) global string loops with length up to 1000 times the core width.

- Global string loops decay within ~1 oscillation period.
- Radiate massive and massless radiation according to initial energies.
- Massive particles are non-relativistic and eventually decay to massless radiation.
- Spectrum of massless radiation is 1/k.

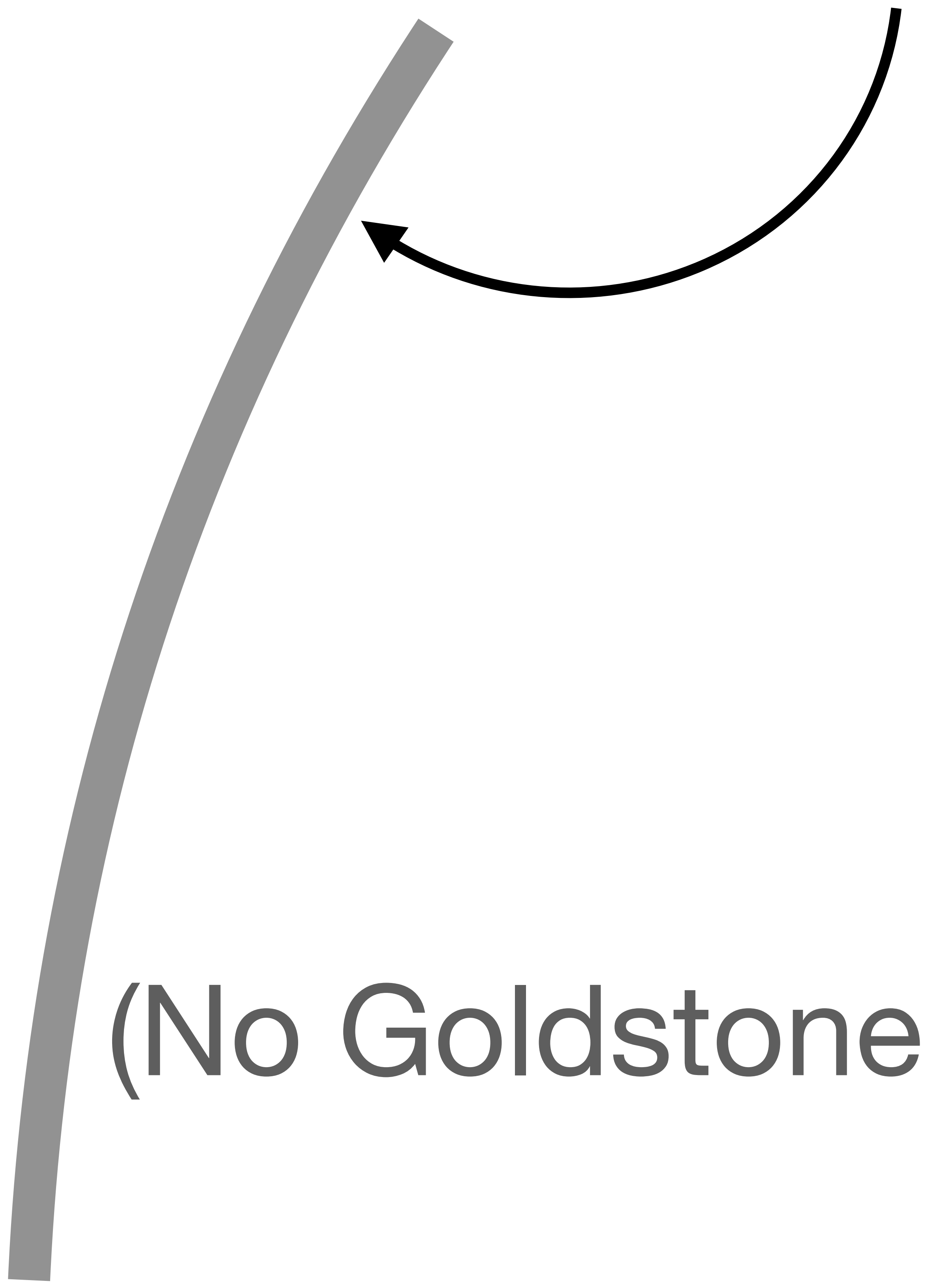
Consistent with Hagmann & Sikivie

- Caveat: Need to extrapolate by many orders of magnitude. Can't detect logarithmic effects.

(Code is available on request.)

Evolution

Gauge strings



$$L = \frac{1}{2}|D_\mu \phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

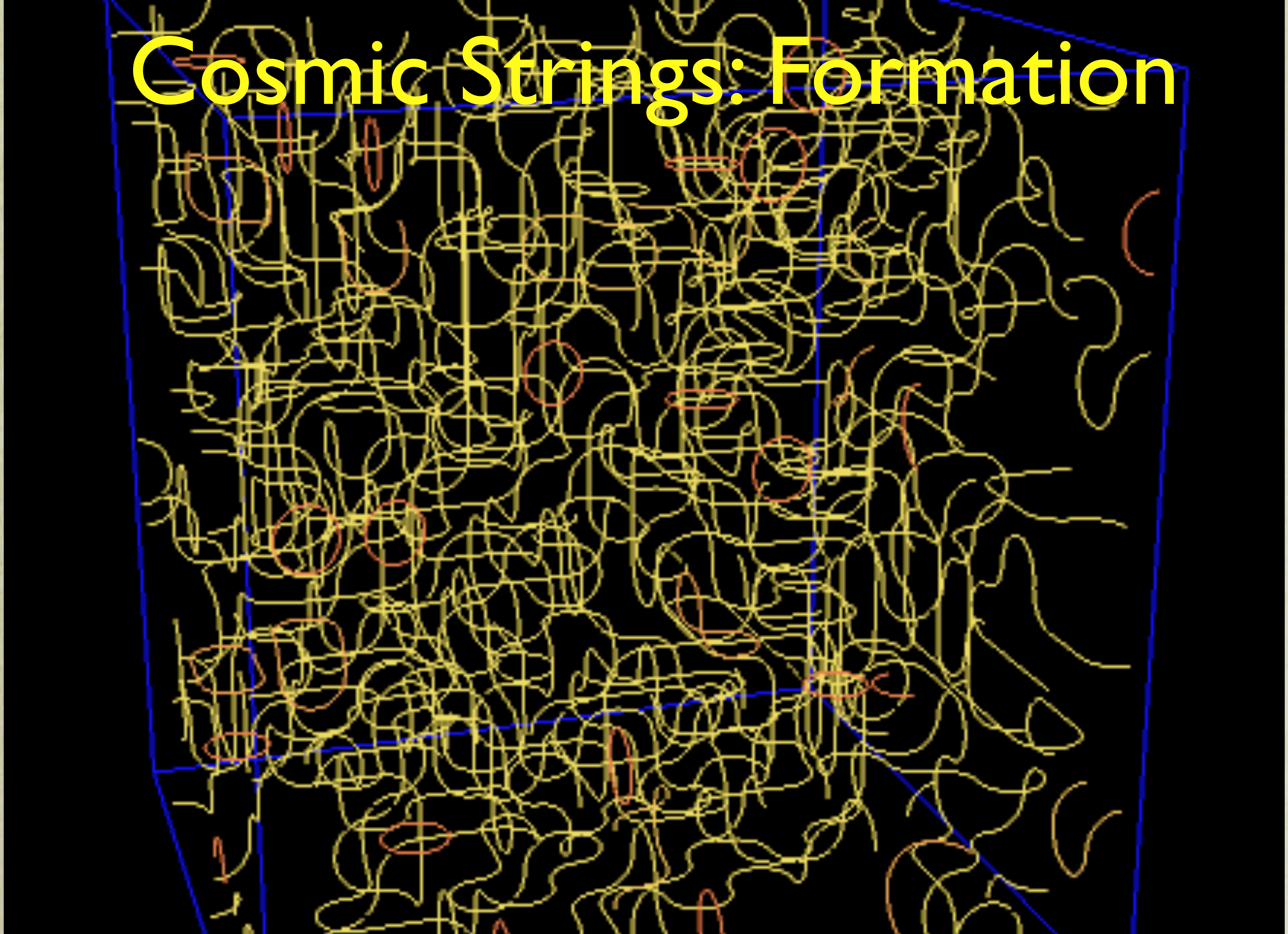
Straight string: $\phi = \eta f(r) e^{i\varphi} \qquad A_i = v(r)\epsilon_{ij}\frac{x^j}{r^2}$

$$f(r) \sim \text{“}\tanh(r/w)\text{”} \qquad v(r) \sim \text{“}\tanh^2(mr)\text{”}$$

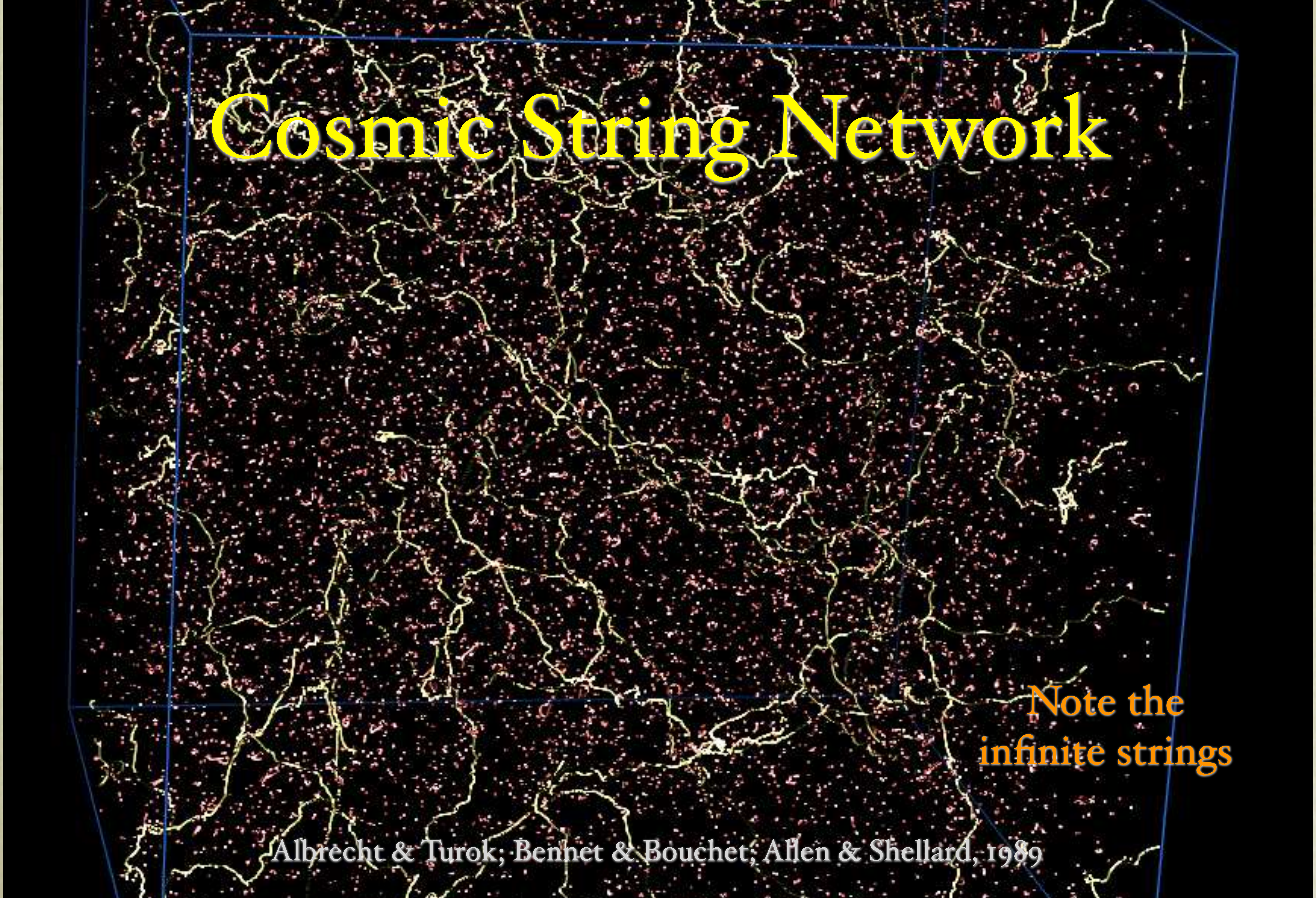
Energy density falls off as exp(-mr) because all fields are massive.

D. Matsunami, L. Pogosian, A. Saurabh, & TV, 2020

Cosmic Strings: Formation



Cosmic String Network



Note the
infinite strings

Albrecht & Turok; Bennet & Bouchet; Allen & Shellard, 1989

Evolution

Gravitational waves or massive radiation?

Nambu-Goto action: $S = -\mu \int d^2\sigma \sqrt{-g_2}$

Loops decay by gravitational radiation. **TV & A. Vilenkin, 1985; …**

Full field theory simulations:

Loops decay by particle radiation. **M. Hindmarsh et al, 2009; …**

Crucial to resolve for experiments (LIGO, NanoGrav,…) looking for gravitational wave signatures.

Evolution

Simulation equations

Technical note: Use Numerical Relativity technique for numerical stability.

$$\partial_t^2 \phi_a = \nabla^2 \phi_a - e^2 A_i A_i \phi_a - 2e\epsilon_{ab}\partial_i \phi_b A_i - e\epsilon_{ab}\phi_b \Gamma$$
$$- \lambda(\phi_b \phi_b - \eta^2)\phi_a$$
$$\partial_t F_{0i} = \nabla^2 A_i - \partial_i \Gamma + e(\epsilon_{ab}\phi_a \partial_i \phi_b + e A_i \phi_a \phi_a)$$
$$\partial_t \Gamma = \partial_i F_{0i} - g_p^2 [\underline{\partial_i F_{0i} + e\epsilon_{ab}\phi_a \partial_t \phi_b}],$$

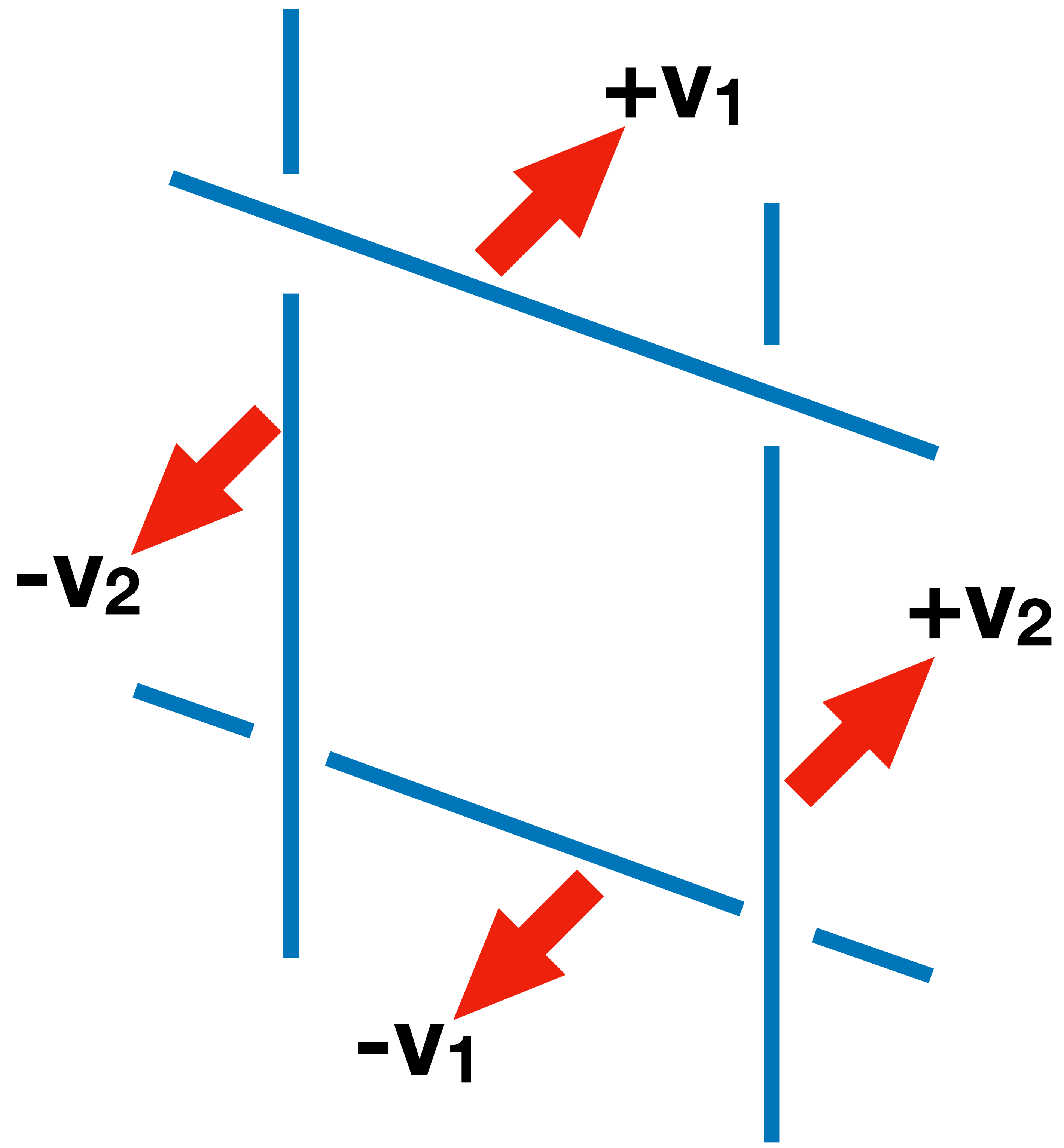
Gauss constraint

$$\Gamma = \partial_i A_i$$

(Code is available on request.)

Evolution

Initial conditions



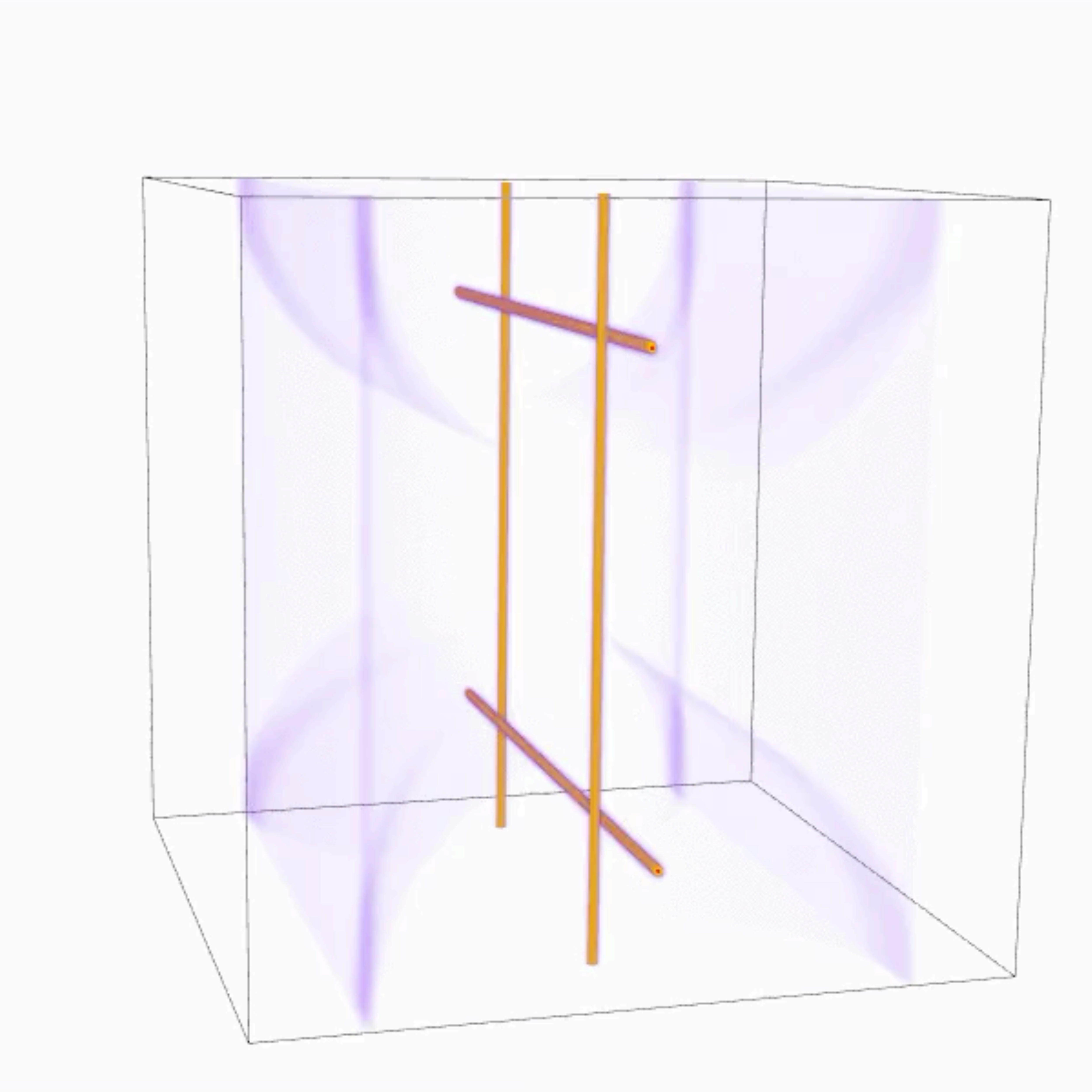
Technical notes

Boost takes the gauge field out of temporal gauge. Then one needs to perform a gauge transformation to go back to temporal gauge.

Periodic boundary conditions require some smoothing functions.

Evolution

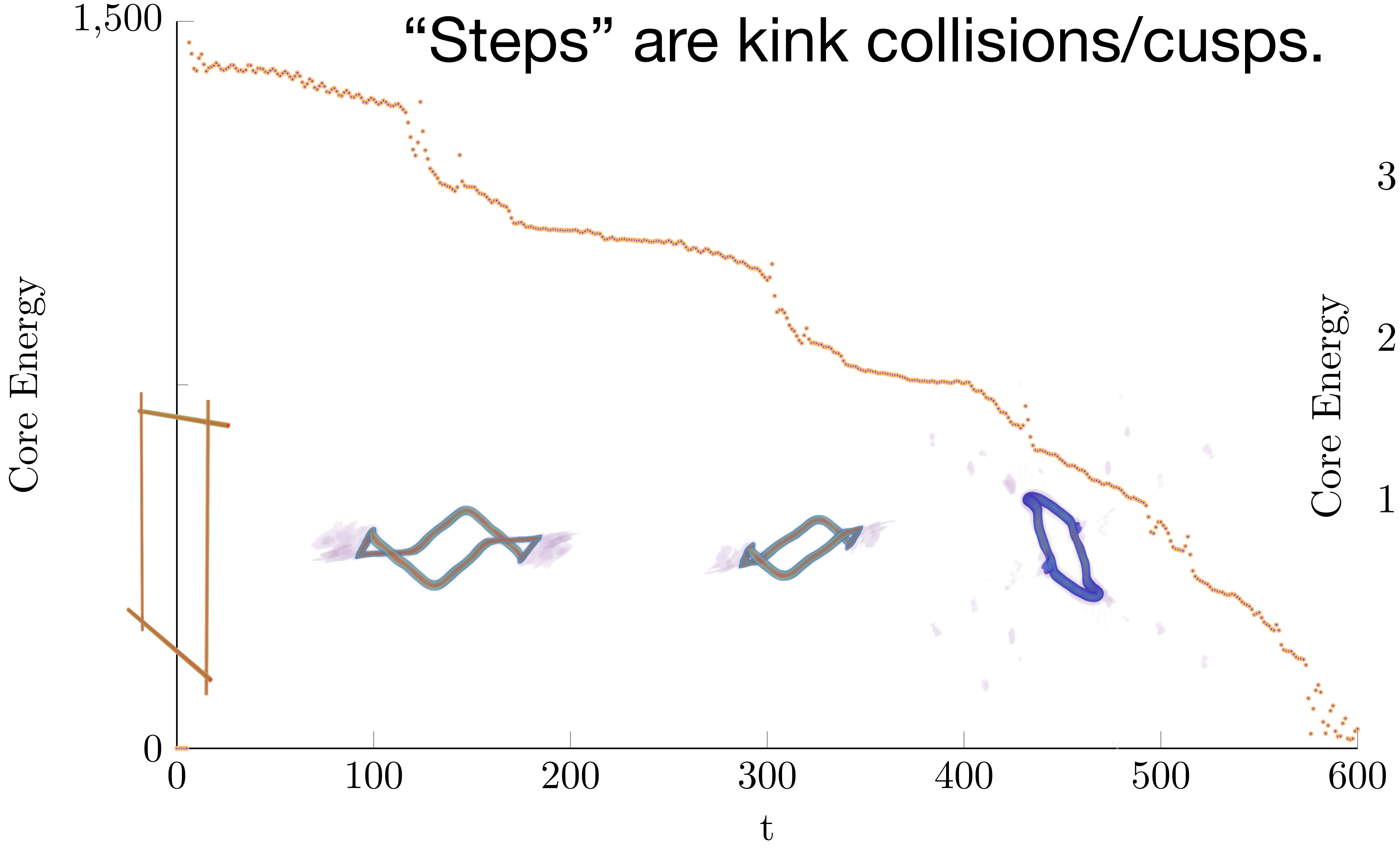
Animation

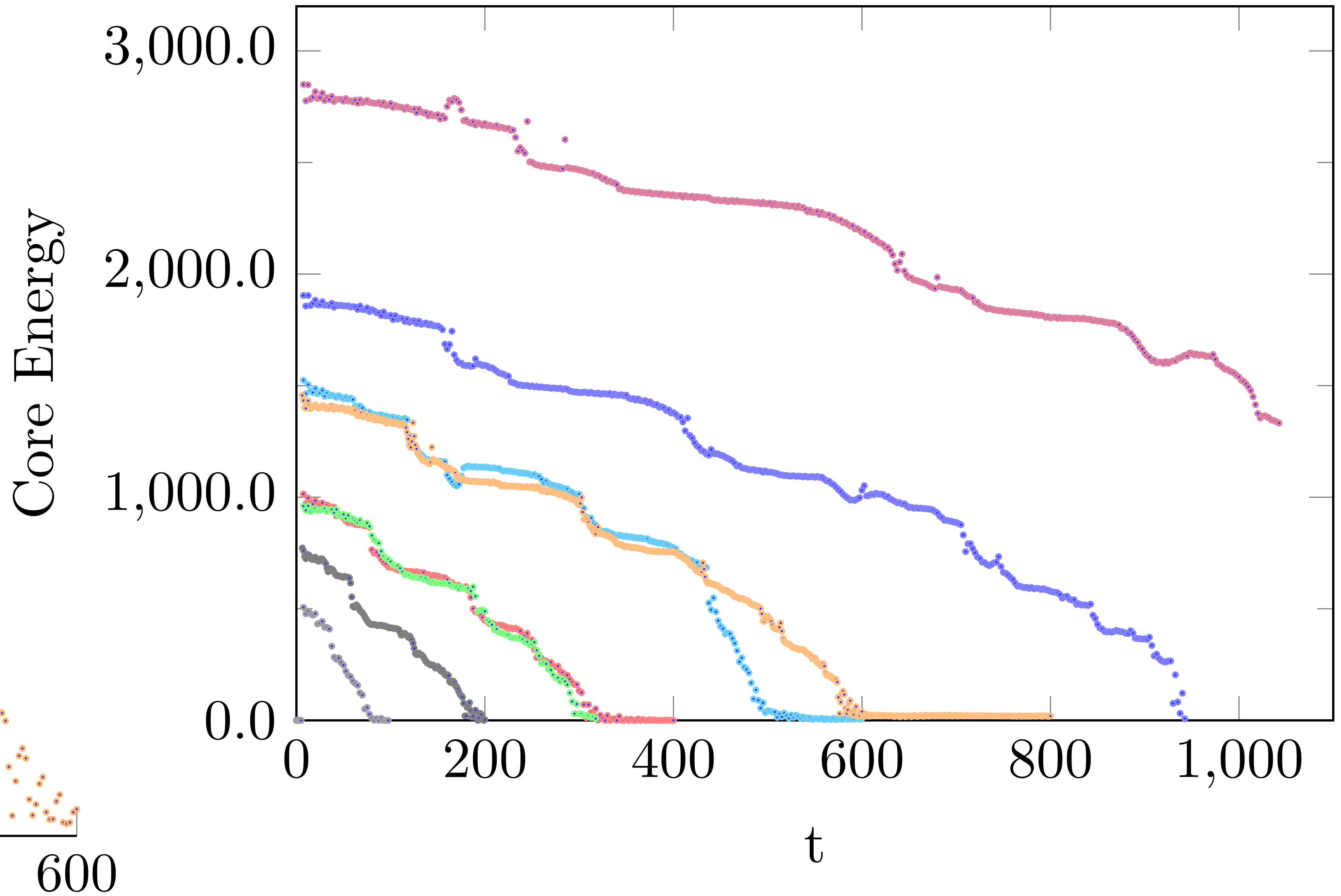


Evolution

Loop energy vs. time

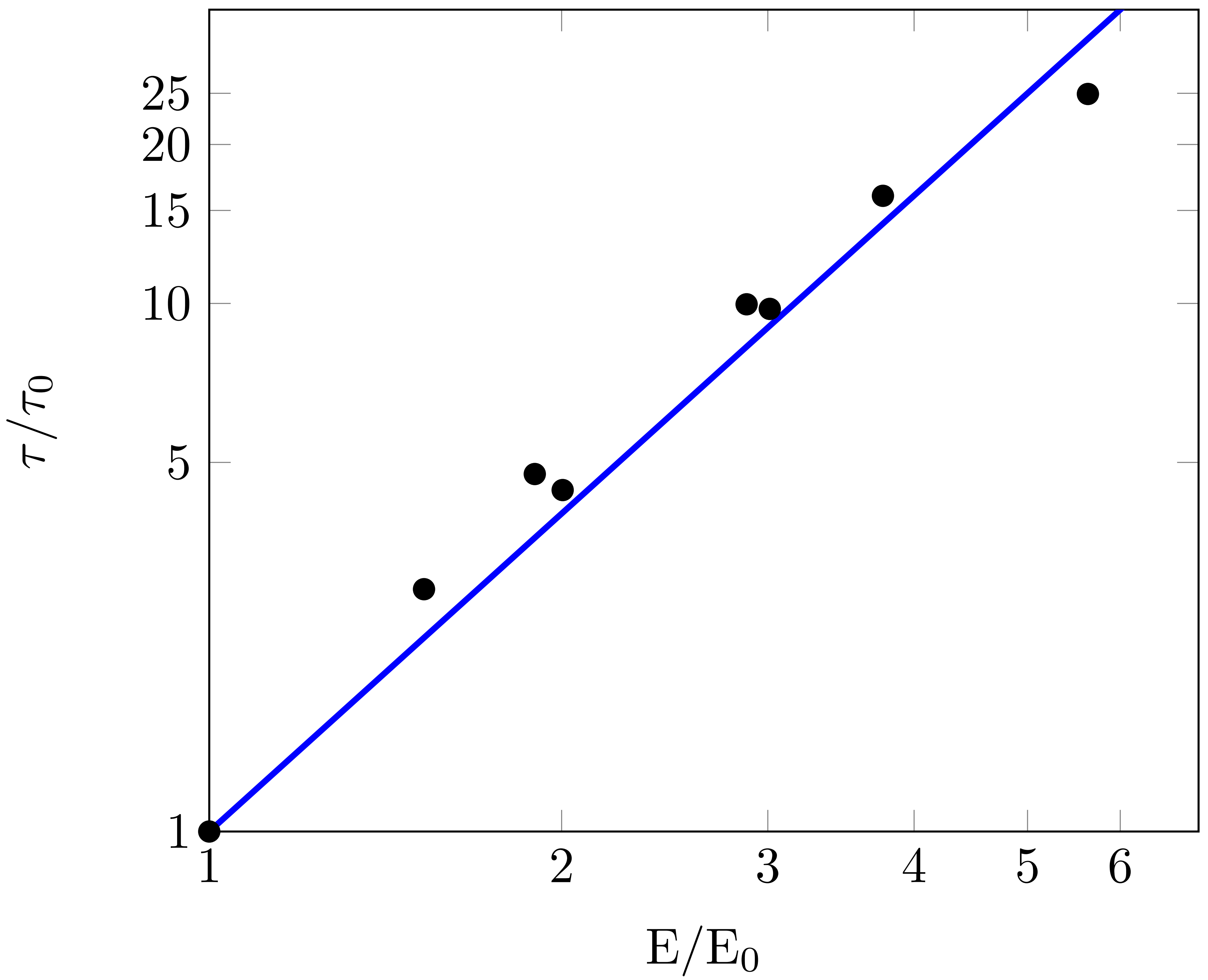
“Steps” are kink collisions/cusps.





Evolution

Lifetime vs. initial length



$$\tau_{\text{particle}} \propto L^2 \qquad \tau_{\text{grav}} \propto L$$

$$\tau_{\text{grav}} < \tau_{\text{particle}} \text{ for large } L$$

$$L_{\text{crit}} \sim \frac{w}{G\mu}$$

where w=width of the string, μ=tension.

Strings with tension above the QCD scale primarily decay by gravitational radiation.

High frequency cutoff on gravitational wave spectrum due to particle radiation.

P. Auclair, D. Steer & TV, 2020

Gravitational radiation

Kinks and cusps

P. Auclair, D. Steer & TV, 2020

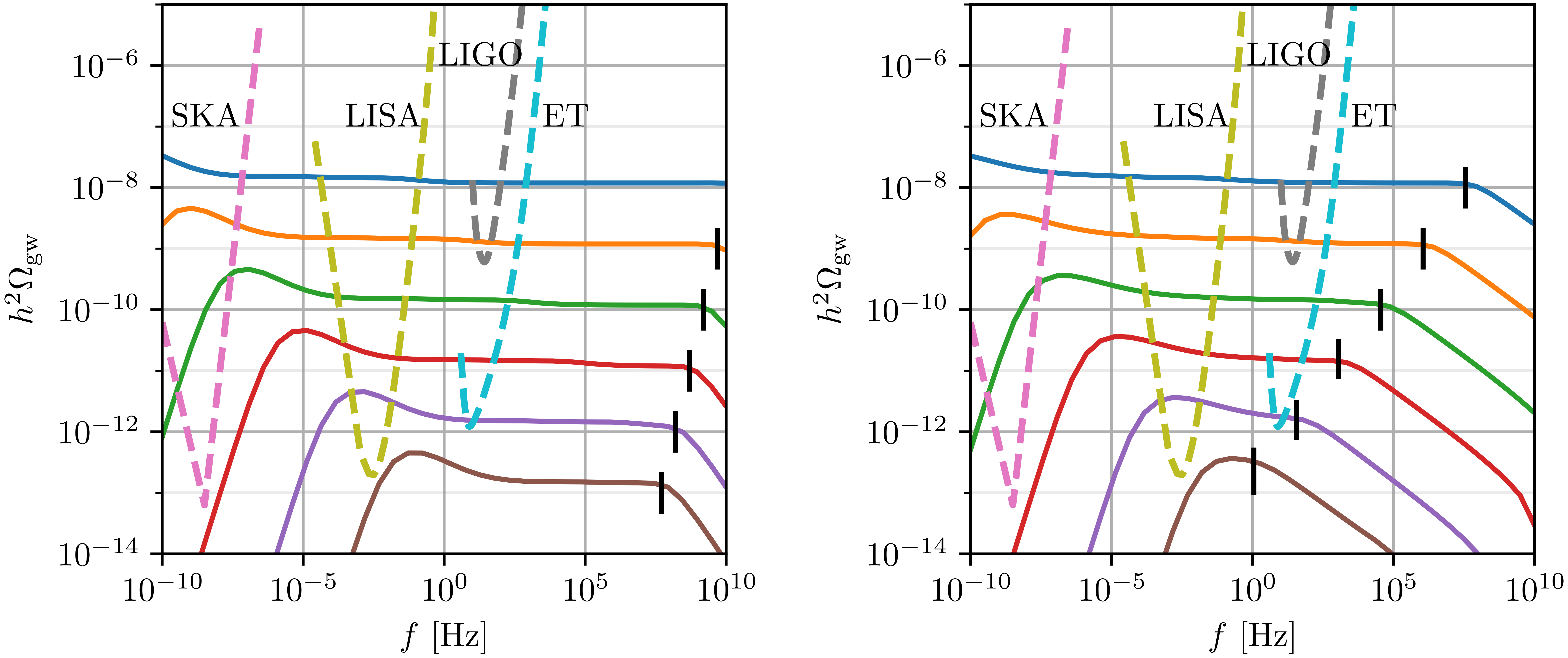


FIG. 3: SBGW including the backreaction of particle emission on the loop distribution. LH panel: kinks on loops, RH panel: cusps on loop. The spectra are cutoff at high frequency, as indicated by the black vertical lines. $G\mu$ ranges from 10^{-17} (lower curve), through 10^{-15}, 10^{-13},10^{-11}, 10^{-9} and 10^{-7} (upper curve). Also plotted are the power-law integrated sensitivity curves from SKA (pink dashed) [44], LISA (yellow dashed) [45], adv-LIGO (grey dashed) [46] and Einstein Telescope (blue dashed) [47, 48].

Conclusion

Formation & Evolution

- ***Formation:*** Universal results for the number density of topological defects formed in a quantum phase transition.

- ***Global string loop evolution:*** Loops decay in about 1 oscillation period, emit massive and massless Goldstone boson radiation. String core appears fluffy, probably due to excitation of bound states on core. Goldstone boson spectrum goes as 1/k and with bump at $m_\chi/2$.

- ***Gauge string loop evolution:*** Loops larger than a critical length w/Gμ, decay primarily to gravitational radiation.