High Energy Behaviour of Form Factors

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With Johannes Henn & Matthias Steinhauser

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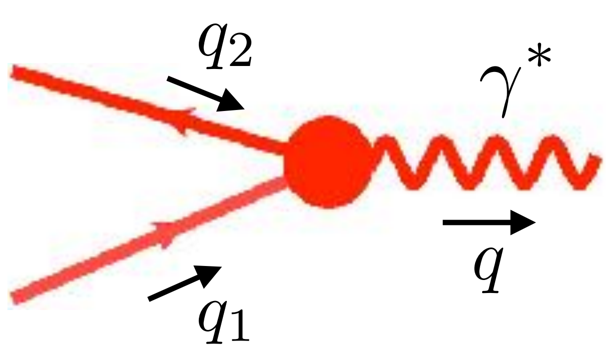
Goal & Motivation

- Infrared divergences: important quantities
- Consider: QCD corrections to photon-quark vertex



$$V^\mu(q_1, q_2) = \bar{v}(q_2)\Gamma^\mu(q_1, q_2)u(q_1)$$

- Vertex function: characterised by two scalar form factors F_1, F_2

$$\Gamma^\mu(q_1, q_2) = Q_q \left[F_1(q^2)\gamma^\mu - \frac{i}{2m} F_2(q^2)\sigma^{\mu\nu} q_\nu \right]$$

- Consider: Form factors of massive quarks
- Important quantities: F_1 is building block for variety of observables

  e.g. Xsection of hadron production in $e^- e^+$ annihilation & derived quantities like forward-backward asymmetry

- Also consider: the massless scenario → F_1

Goal & Motivation

- State-of-the-art results

  $m \neq 0$ F_1, F_2 at 3-loop [Henn, Smirnov, Smirnov, Steinhauser '16]
  $m = 0$ F_1 at 4-loop
  } in large N_c limit in $SU(N_c)$
  [Henn, Smirnov, Smirnov, Steinhauser, Lee '16]
  [Manteuffel, Schabinger '16]

- Next steps: compute the full results for general N_c

  $\rightsquigarrow$ underway by several groups

- We address: What can we say about next order?

  $\rightsquigarrow$ indeed, IR poles can be predicted (partially) by exploiting RG evolution of FF

  RESULTS

  $m \neq 0 \rightsquigarrow F_1$ at 4-loop in large N_c and high energy limit upto $1/\epsilon^2$

  $m = 0 \rightsquigarrow F_1$ at 5-loop in large N_c and high energy limit upto $1/\epsilon^3$

- We also obtain process independent functions relating massive & massless amplitudes in high-energy limit at 3 & 4-loops RESULTS

GOAL Exploit RG evolution of FF

Plan Of The Talk

- RG evolution: massive
  - Cute technique to solve
- RG evolution: massless
- Process independent functions
- Conclusions

RG Equation: Massive

[Sudakov '56; Mueller '79; Collins '80; Sen '81]
[Magnea, Sterman '90]
[Gluza, Mitov, Moch, Riemann '07, '09]

- FF satisfies KG eqn in dimensional reg.

$$-\frac{d}{d\ln\mu^2}\ln\tilde{F}\left(\hat{a}_s,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2},\epsilon\right)=\frac{1}{2}\left[\tilde{K}\left(\hat{a}_s,\frac{m^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)+\tilde{G}\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)\right]$$

⟺ QCD factorisation, gauge & RG invariance

- The form factor

$$F=Ce^{\ln\tilde{F}}$$

Matching coefficient

$Q^2=-q^2=-(p_1+p_2)^2$
$d=4-2\epsilon$
$\hat{a}_s\equiv\hat{\alpha}_s/4\pi$
μ : scale to keep $\hat{a}_s$ dimensionless
μ_R : renormalisation scale

- Goal: Solve the RG
- Strategy: Use bare coupling $\hat{a}_s$ instead of renormalised one a_s

[Ravindran '06: For Massless]

SOLVING RG EQUATION: MASSIVE

RG invariance of FF wrt μ_R

$$\frac{d}{d\ln\mu_R^2}\tilde{K}\left(\hat{a}_s,\frac{m^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)=-\frac{d}{d\ln\mu_R^2}\tilde{G}\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)=-A\left(a_s\left(\mu_R^2\right)\right)$$

Cusp anomalous dimension

$$\tilde{K}\left(\hat{a}_s,\frac{m^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)=K\left(a_s\left(m^2\right),\epsilon\right)-\int_{m^2}^{\mu_R^2}\frac{d\mu_R^2}{\mu_R^2}A\left(a_s\left(\mu_R^2\right)\right)$$

$$\tilde{G}\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)=G\left(a_s\left(Q^2\right),\epsilon\right)+\int_{Q^2}^{\mu_R^2}\frac{d\mu_R^2}{\mu_R^2}A\left(a_s\left(\mu_R^2\right)\right)$$

Boundary terms

SOLVING RG EQUATION: MASSIVE

Initial goal: Solve for $\ln \tilde{F}$ in powers of bare $\hat{a}_s$

→ Need all quantities in powers of $\hat{a}_s$

Expand

$$\mathcal{B} \in \{K, G, A\}$$

$$\mathcal{B}\left(a_s\left(\lambda^2\right)\right) \equiv \sum_{k=1}^{\infty} a_s^k\left(\lambda^2\right) \mathcal{B}_k$$

$$\lambda \in \{m, Q, \mu_R\}$$

Renormalisation constant

Use

$$\hat{a}_s = a_s(\mu_R^2) Z_{a_s}\left(\mu_R^2\right)\left(\frac{\mu^2}{\mu_R^2}\right)^{-\epsilon}$$

$$Z_{a_s}^{-1}(\lambda^2) = 1 + \sum_{k=1}^{\infty} \hat{a}_s^k \left(\frac{\lambda^2}{\mu^2}\right)^{-k\epsilon} \hat{Z}_{a_s}^{-1,(k)}$$

functions of β_i, ϵ

Expansion of $\mathcal{B}$ in powers of $\hat{a}_s$

Solving RG Equation: Massive

Soln of $\mathcal{B}$ in powers of $\hat{a}_s$

$$\mathcal{B}\left(a_s\left(\lambda^2\right)\right) = \sum_{k=1}^{\infty} \hat{a}_s^k \left(\frac{\lambda^2}{\mu^2}\right)^{-k\epsilon} \hat{\mathcal{B}}_k$$

with

$$\hat{\mathcal{B}}_1 = \mathcal{B}_1\,,$$

$$\hat{\mathcal{B}}_2 = \mathcal{B}_2 + \mathcal{B}_1 \hat{Z}_{a_s}^{-1,(1)}\,,$$

$$\hat{\mathcal{B}}_3 = \mathcal{B}_3 + 2\mathcal{B}_2 \hat{Z}_{a_s}^{-1,(1)} + \mathcal{B}_1 \hat{Z}_{a_s}^{-1,(2)}\,,$$

$$\hat{\mathcal{B}}_4 = \mathcal{B}_4 + 3\mathcal{B}_3 \hat{Z}_{a_s}^{-1,(1)} + \mathcal{B}_2 \left\{ \left(\hat{Z}_{a_s}^{-1,(1)}\right)^2 + 2\hat{Z}_{a_s}^{-1,(2)} \right\} + \mathcal{B}_1 \hat{Z}_{a_s}^{-1,(3)}$$

and so on...

The integral becomes a polynomial integral $\rightsquigarrow$ trivial

$$\int_{\lambda^2}^{\mu_R^2} \frac{d\mu_R^2}{\mu_R^2} A\left(a_s\left(\mu_R^2\right)\right) = \sum_{k=1}^{\infty} \hat{a}_s^k \frac{1}{k\epsilon} \left[\left(\frac{\lambda^2}{\mu^2}\right)^{-k\epsilon} - \left(\frac{\mu_R^2}{\mu^2}\right)^{-k\epsilon} \right] \hat{A}_k$$

Un-renormalised Solution: Massive

Solution of KG in powers of bare $\hat{a}_s$

$$\ln \tilde{F}\left(\hat{a}_s, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \epsilon\right) = \sum_{k=1}^{\infty} \hat{a}_s^k \left[\left(\frac{Q^2}{\mu^2}\right)^{-k\epsilon} \hat{\tilde{\mathcal{L}}}_k^Q(\epsilon) + \left(\frac{m^2}{\mu^2}\right)^{-k\epsilon} \hat{\tilde{\mathcal{L}}}_k^m(\epsilon)\right]$$

with

$$\hat{\tilde{\mathcal{L}}}_k^Q(\epsilon) = -\frac{1}{2k\epsilon}\left[\hat{G}_k + \frac{1}{k\epsilon}\hat{A}_k\right],$$

$$\hat{\tilde{\mathcal{L}}}_k^m(\epsilon) = -\frac{1}{2k\epsilon}\left[\hat{K}_k - \frac{1}{k\epsilon}\hat{A}_k\right]$$

Renormalised Solution

$$\hat{a}_s = a_s(\mu_R^2) Z_{a_s}(\mu_R^2)\left(\frac{\mu^2}{\mu_R^2}\right)^{-\epsilon}$$

$$= \sum_{k=1}^{\infty}\left[a_s^k(Q^2)\tilde{\mathcal{L}}_k^Q + a_s^k(m^2)\tilde{\mathcal{L}}_k^m\right]$$

To obtain the renormalised solution in powers of general $a_s(\mu_R^2)$

$\rightsquigarrow$ use d-dimensional evolution of $a_s(\mu_R^2)$

$$\frac{d}{d\ln\mu_R^2} a_s(\mu_R^2) = -\epsilon a_s(\mu_R^2) - \sum_{k=0}^{\infty} \beta_k a_s^{k+2}(\mu_R^2)$$

Solved iteratively

Renormalised Solution: Massive

Renormalised Solution

$$\ln \tilde{F} = \sum_{k=1}^{\infty} a_s^k(\mu_R^2)\tilde{\mathcal{L}}_k$$

For $\mu_R^2 = m^2$ at one loop

$$\tilde{\mathcal{L}}_1 = \frac{1}{\epsilon}\Bigg\{-\frac{1}{2}\Bigg(G_1 + K_1 - A_1 L\Bigg)\Bigg\} + \frac{L}{2}\Bigg(G_1 - \frac{A_1\,L}{2}\Bigg) - \epsilon\Bigg\{\frac{L^2}{4}\Bigg(G_1 - \frac{A_1 L}{3}\Bigg)\Bigg\}$$
$$+\,\epsilon^2\Bigg\{\frac{L^3}{12}\Bigg(G_1 - \frac{A_1\,L}{4}\Bigg)\Bigg\} - \epsilon^3\Bigg\{\frac{L^4}{48}\Bigg(G_1 - \frac{A_1 L}{5}\Bigg)\Bigg\} + \epsilon^4\Bigg\{\frac{L^5}{240}\Bigg(G_1 - \frac{A_1 L}{6}\Bigg)\Bigg\} + \mathcal{O}(\epsilon^5)$$

At two loop

$$\tilde{\mathcal{L}}_2 = \frac{1}{\epsilon^2}\Bigg\{\frac{\beta_0}{4}\Bigg(G_1 + K_1 - A_1 L\Bigg)\Bigg\} - \frac{1}{\epsilon}\Bigg\{\frac{1}{4}\Bigg(G_2 + K_2 - A_2 L\Bigg)\Bigg\} + \frac{L}{2}\Bigg(G_2 - \frac{A_2 L}{2}\Bigg)$$
$$-\frac{\beta_0 L^2}{4}\Bigg(G_1 - \frac{A_1 L}{3}\Bigg) - \epsilon\Bigg\{\frac{L^2}{2}\Bigg(G_2 - \frac{A_2 L}{3}\Bigg) - \frac{\beta_0 L^3}{4}\Bigg(G_1 - \frac{A_1 L}{4}\Bigg)\Bigg\}$$
$$+\,\epsilon^2\Bigg\{\frac{L^3}{3}\Bigg(G_2 - \frac{A_2 L}{4}\Bigg) - \frac{7\beta_0 L^4}{48}\Bigg(G_1 - \frac{A_1 L}{5}\Bigg)\Bigg\} - \epsilon^3\Bigg\{\frac{L^4}{6}\Bigg(G_2 - \frac{A_2 L}{5}\Bigg)$$
$$-\frac{\beta_0 L^5}{16}\Bigg(G_1 - \frac{A_1 L}{6}\Bigg)\Bigg\} + \mathcal{O}(\epsilon^4)$$

and so on...

$$L = \log(Q^2/m^2)$$

New Results: Massive

- Conformal theory $\beta_i = 0$: all order result

$$\tilde{\mathcal{L}}_k = \sum_{l=0}^{\infty} (-\epsilon k)^{l-1} \frac{L^l}{2\, l!} \left(G_k + \delta_{0l} K_k - \frac{A_k L}{l+1} \right)$$

- Form Factor

$$F = C\left(a_s\left(m^2\right), \epsilon\right) e^{\ln \tilde{F}}$$ → consistent with literature up to 3-loop

[Gluza, Mitov, Moch, Riemann '07, '09]

- State-of-the-art results

F_1, F_2 at 3-loop in large N_c [Henn, Smirnov, Smirnov, Steinhauser '16]

- New results in 1704.07846

F_1 at 4-loop in large N_c and high energy limit

upto $\frac{1}{\epsilon^2}$

F_2 is suppressed by m^2/q^2 in high energy limit

Determining Unknown Constants: Massive

Determining unknown constants G, K, C in large N_c limit

Comparing with explicit computations

- ★ G_1 to $\mathcal{O}(\epsilon^2)$, G_2 to $\mathcal{O}(\epsilon)$ [Gluza, Mitov, Moch, Riemann '07 '09]

  G_3 to $\mathcal{O}(\epsilon^0)$ new! ⟷ F_1 at 3-loop [Henn, Smirnov, Smirnov, Steinhauser '16] [Gluza, Mitov, Moch, Riemann '09]

- ★ K_1, K_2

  K_3 new!

- ★ C_1 to $\mathcal{O}(\epsilon^2)$, C_2 to $\mathcal{O}(\epsilon)$ [Gluza, Mitov, Moch, Riemann '09]

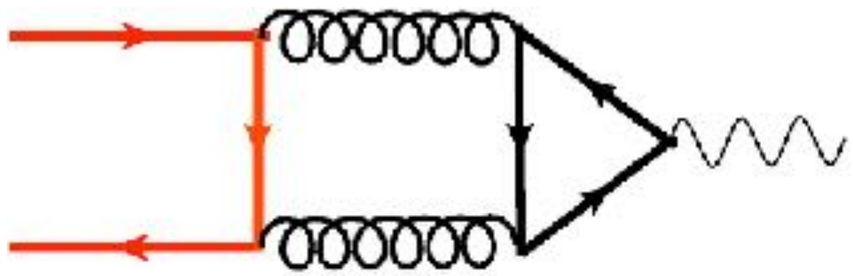
  C_1 to $\mathcal{O}(\epsilon^4)$, C_2 to $\mathcal{O}(\epsilon^2)$, C_3 to $\mathcal{O}(\epsilon^0)$ new!

  explicit computation

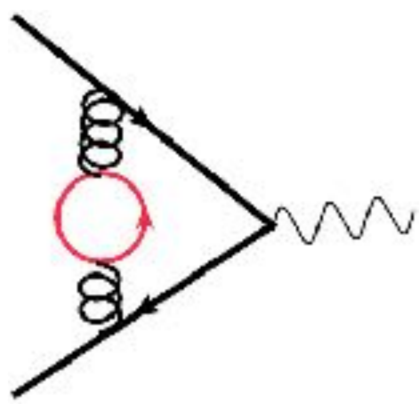
- ★ A_4 became available recently [Henn, Smirnov, Smirnov, Steinhauser '16] [Henn, Smirnov, Smirnov, Steinhauser, Lee '16]

COMMENTS: MASSIVE

- Excludes singlet contributions



- Excludes closed heavy-quark loops



Obey similar exponentiation

[Kühn, Moch, Penin, Smirnov '01]
[Feucht, Kühn, Moch '03]

⇝ Sub-leading in large N_c limit

⇝ Hence, we have not considerer these

Massless Scenario

RG Equation: Massless

- FF satisfies KG eqn

$$-\frac{d}{d\ln\mu^2}\ln\tilde{F}\left(\hat{a}_s,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2},\epsilon\right)=\frac{1}{2}\left[\tilde{K}\left(\hat{a}_s,\frac{m^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)+\tilde{G}\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\epsilon\right)\right]$$

[Sudakov '56; Mueller '79; Collins '80; Sen '81]

Solved exactly the similar way [Ravindran '06]

$$\ln\tilde{F}\left(\hat{a}_s,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2},\epsilon\right)=\sum_{k=1}^{\infty}\hat{a}_s^k\left[\left(\frac{Q^2}{\mu^2}\right)^{-k\epsilon}\hat{\tilde{\mathcal{L}}}_k^Q(\epsilon)+\left(\frac{m^2}{\mu^2}\right)^{-k\epsilon}\hat{\tilde{\mathcal{L}}}_k^m(\epsilon)\right]$$

Up to 4-loop: present [Moch, Vermaseren, Vogt '05] [Ravindran '06]

5-loop solution

new!

RG Equation: Massless

- Conformal theory $\beta_i = 0$: all order result

$$\hat{\hat{\mathcal{L}}}_k^Q = \frac{1}{\epsilon^2}\left\{ - \frac{1}{2k^2} A_k \right\} + \frac{1}{\epsilon}\left\{ - \frac{1}{2k} G_k \right\}$$

[Bern, Dixon, Smirnov '05]

[TA, Banerjee, Dhani, Rana, Ravindran, Seth '17]

- FF

$$F = C e^{\ln \tilde{F}}$$

Matching coefficient = 1

- State-of-the-art results

F at 4-loop in large N_c [Henn, Smirnov, Smirnov, Steinhauser, Lee '16]

- New results in 1704.07846

F at 5-loop in large N_c and high energy limit

upto $\frac{1}{\epsilon^3}$

Determining Unknown Constants: Massless

Determining unknown constants in large N_c limit

Comparing with explicit computations

★ G_1 to $\mathcal{O}(\epsilon^6)$, G_2 to $\mathcal{O}(\epsilon^4)$, G_3 to $\mathcal{O}(\epsilon^2)$

[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser '09]
[Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]

G_4 to $\mathcal{O}(\epsilon^0)$ new! ⟷ F at 4-loop

[Henn, Smirnov, Smirnov, Steinhauser, Lee '16]

★ $K_i = K_i(A_k, \beta_k)$ do not appear in the final expressions

$\rightsquigarrow$ get cancelled against similar terms arising from G

Comments: Massive & Massless

★ G are same for massive and massless [Mitov, Moch '07]

↪ expected! Governed by universal cusp AD

↪ Manifestly clear in our methodology

$$\tilde{G}\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon\right) = G\left(a_s\left(Q^2\right), \epsilon\right) + \int_{Q^2}^{\mu_R^2} \frac{d\mu_R^2}{\mu_R^2} A\left(a_s\left(\mu_R^2\right)\right)$$

★ For massive K_i enter only into the poles of $\tilde{\mathcal{L}}_k$

⇝ Constants and $\mathcal{O}(\epsilon^k)$ terms can be determined from massless calculation

⇝ could lead to deeper understanding of the connection between massive & massless FF

PROCESS INDEPENDENT FUNCTION

- QCD factorisation: massive amplitudes shares essential properties with the corresponding massless ones in the high-energy limit

$$\mathcal{M}^{(m)} = \prod_{i \in \{\text{all legs}\}} \left[Z_{[i]}^{(m|0)} \left(\frac{m^2}{\mu^2} \right) \right]^{1/2} \mathcal{M}^{(0)}$$

[Moch, Mitov '07]

Massive

Massless

Universal and depends only on the external partons!

- Can be computed using simplest amplitudes: FF

$$Z_{[q]}^{(m|0)} = \frac{F(Q^2, m^2, \mu^2)}{\overline{F}(Q^2, \mu^2)}$$

★ Q^2 independence is manifestly clear: governed by G, same for massive & massless FF

★ $\mathcal{O}(\epsilon^0)$ at 3-loop, upto $\mathcal{O}(1/\epsilon^2)$ at 4-loop $\rightsquigarrow$ new!

★ Relates dimensionally regularised amplitudes to those where the IR divergence is regularised with a small quark mass.

Conclusions

- ★ RG equations governing massive & massless quark-photon FF are discussed.

- ★ Elegant derivation for analytic solution is proposed

  → key idea: use bare coupling

- ★ Q^2 dependence is governed by G & cusp AD: same for massive & massless

- ★ Massive: non-trivial matching coefficient C

- ★ Massive: F_1 at 4-loop in large N_c and high energy limit to $\frac{1}{\epsilon^2}$

  Massless: F at 5-loop in large N_c and high energy limit to $\frac{1}{\epsilon^3}$

THANK YOU!