Superfluid Dark Matter: Beyond the Dichotomy of Dark Matter vs. Modified Gravity

Tobias Mistele
in collaboration with Sabine Hossenfelder

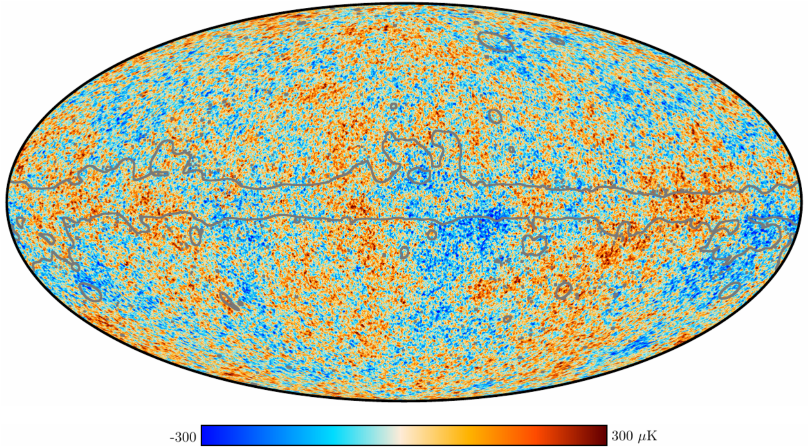
Frankfurt Institute for Advanced Studies (FIAS)

IIT Hyderabad
June 18, 2021

FIAS Frankfurt Institute for Advanced Studies

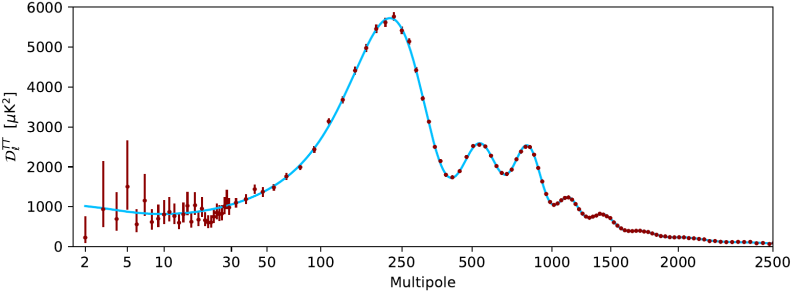
Phases of “dark matter”

Cosmic Microwave Background



[Planck 2018]

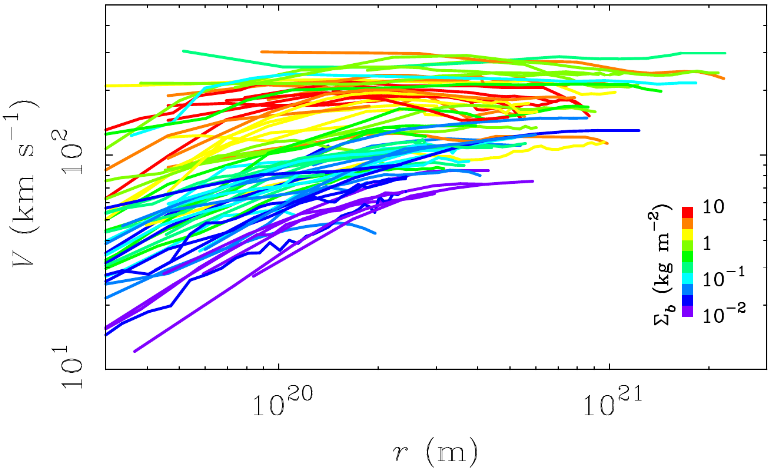
Cosmic Microwave Background



[Planck 2018]

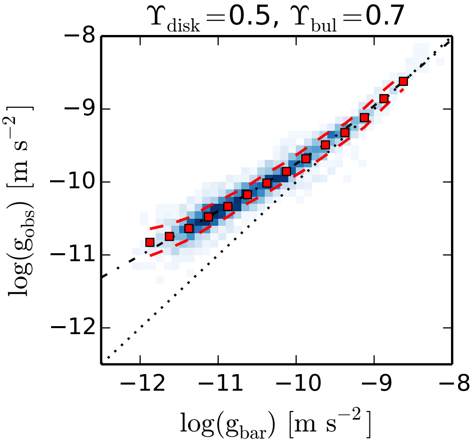
- Simple explanation: collisionless dark fluid
- Without dark fluid: No simple explanation (e.g. for 2nd/3rd peak ratio)

Galaxies - Rotation Curves



[Famaey, McGaugh 2012]

Galaxies - Radial Acceleration Relation (RAR)



[Lelli et al. 2016]

- Simple explanation: MOND

$$g_{\rm obs} = g_{\rm bar}\, \nu(g_{\rm bar}/a_0)$$

- LCDM: Galaxy formation simulations can maybe reproduce RAR *[e.g. Keller et al. 2017, Navarro et al. 2017]*
- Requires complicated baryonic physics, empirical models

Phases of dark matter?

- Two different regimes: Simple explanation in terms of ΛCDM on cosmological scales, in terms of MOND on galactic scales
- Is there an explanation in terms of different phases of a single underlying substance?
- Superfluid Dark Matter + other hybrid models, e.g. recent model by Skordis & Złośnik

Brief review of SFDM

Warm-up: Superfluids in field theory

- Complex scalar field $\phi = \frac{\rho}{\sqrt{2}} e^{-i\theta}$

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 |\phi|^2 - \lambda_4 |\phi|^4$$

- Has $U(1)$ symmetry $\theta \to \theta + \mathrm{const}$
- Equilibrium: Symmetry $\leftrightarrow$ chemical potential μ
- $H \to H - \mu Q$. At Lagrangian level: $\dot{\theta} \to \dot{\theta} + \mu$
- Effective potential:

$$V_{\mathrm{eff}}(\rho) = \frac{1}{2}(m^2 - \mu^2)\rho^2 + \frac{1}{4}\lambda_4 \rho^4$$

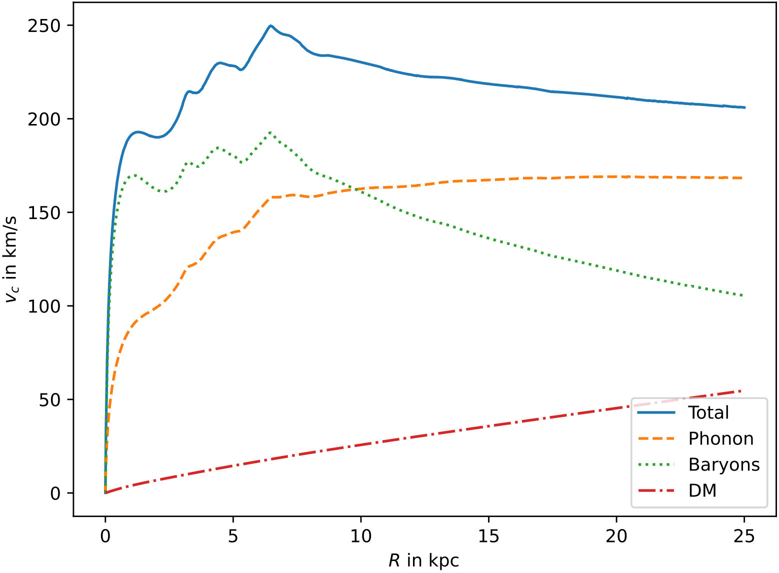
- Condensation for $\mu > m$
- Non-relativistic: $\mu = m + \mu_{\mathrm{nr}}$ with $\mu_{\mathrm{nr}} \ll m$
- Low-energy perturbations: Phonons with dispersion relation

$$\omega = c_s k \, , \, c_s \approx \sqrt{\frac{\mu_{\mathrm{nr}}}{m}} \ll 1 \quad \xRightarrow[\textit{dispersion}]{\textit{linear}} \quad \text{Frictionless flow}$$

Superfluid Dark Matter *[Berezhiani, Khoury 2015]*

- Cosmological scales: Cold Dark Matter particle, $m \sim \mathrm{eV}$
- Galactic scales: Superfluid core
    - Condensate
    - Phonon field mediates a MOND-like force
    - Cored dark matter profile from superfluid
- Galactic scales: Larger radii
    - Superfluid not in equilibrium
    - Match to NFW profile
    - No phonon force

Superfluid Dark Matter: Superfluid core



Superfluid Dark Matter: Superfluid core

- Phonon field θ has effective MOND-like kinetic term and MOND-like coupling to baryons:

$$\mathcal{L} = \frac{2\Lambda}{3}(2m)^{3/2}\sqrt{|X - \beta Y|}X - \lambda\rho_b\,\theta\,,$$
$$X = \dot{\theta} + \hat{\mu} - (\vec{\nabla}\theta)^2/(2m)\,, \quad Y = \dot{\theta} + \hat{\mu}\,, \quad \hat{\mu} = \mu_{\rm nr} - m\phi_{\rm N}$$

  Static MOND limit has $\mathcal{L} \sim X^{3/2}$:

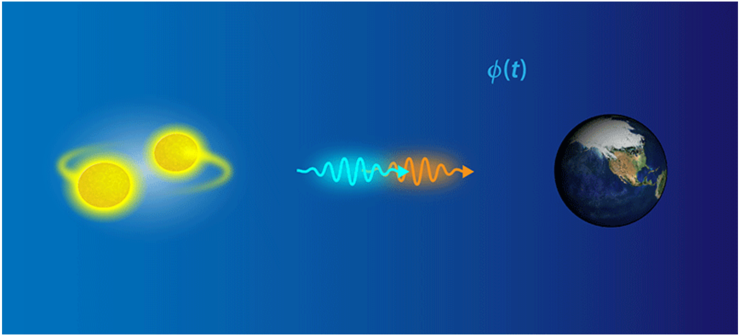
$$(\vec{\nabla}\theta)^2 \gg 2m\hat{\mu}$$

- Total acceleration in MOND limit:

$$\begin{aligned} g_{\rm tot} &= g_{\rm bar} + g_\theta + g_{\rm SF} \\ &\approx g_{\rm bar} + \sqrt{a_0\, g_{\rm bar}} + g_{\rm SF} \end{aligned}$$

How to test?

Constraint from gravitational waves

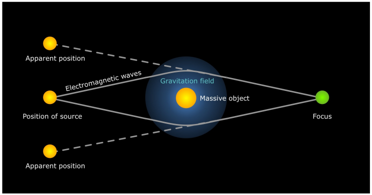


[APS/Alan Stonebraker]

- GW170817/GRB170817A: Electromagnetic and gravitational waves arrive at roughly the same time *[LIGO, VIRGO 2017]*
- No additional force acting on photons *[Sanders 2018]*, *[Boran et al. 2018]*
- E.g. SFDM's phonon force should act only on baryons

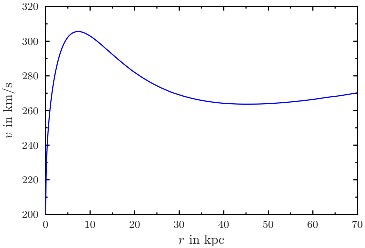
Constraint from gravitational waves

Lensing



[dlr.de, GFDL]

Rotation curves



$\downarrow$

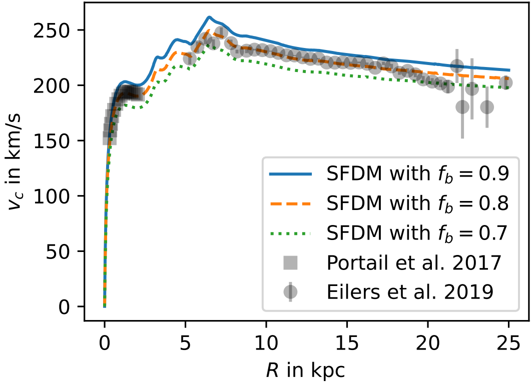
Gravitational pull only

$\downarrow$

Gravitational pull + additional force

- Consistent with strong lensing + kinematic data?
- We checked: Can fit velocity dispersion and Einstein radii simultaneously $\rightarrow$ no challenge for SFDM
  [Hossenfelder, TM 2019]

Milky Way rotation curve *[Hossenfelder, TM 2020]*



- $\sim 20\%$ less baryonic mass than standard MOND
- Superfluid core size: $\sim 65\,\mathrm{kpc}$

Theoretical issues?

[TM 2021]

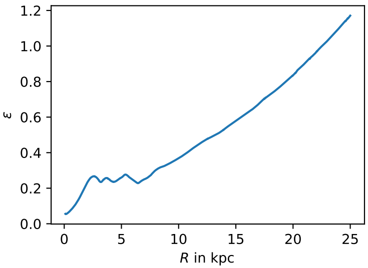
Three problems of SFDM: The stability problem

- Finite-temperature effects parametrized by β required
- Reason: Perturbations $\theta \to \theta + \delta$ in galaxies are unstable

$$\mathcal{L}_{\rm pert}|_{\beta=0} = -\frac{\Lambda m^2}{|\vec{\nabla}\theta|}\dot{\delta}^2 + \dots$$

- But: Both the value of β and the form of the corrections are ad-hoc. Not clear if they follow from any $T = 0$ Lagrangian.

Three problems of SFDM: The MOND limit problem

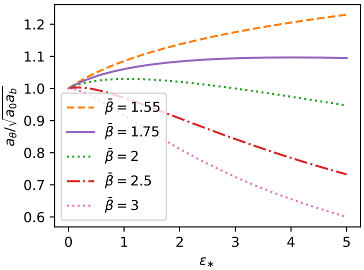


- MOND-like equation for θ if

$$\varepsilon \equiv (2m\hat{\mu})/(\vec{\nabla}\theta)^2 \ll 1$$

- Easily violated, see plot for MW

$\Rightarrow$ Many galaxies: No proper MOND limit



- Pseudo-MOND limit for $\beta \approx 2$: Roughly MOND-like rotation curves for isolated galaxies.
- But: Relies on detail of ad-hoc finite-temperature corrections + lose e.g. standard MOND External Field Effect

Three problems of SFDM: The equilibrium problem

- Superfluid's chemical potential $\leftrightarrow$ $U(1)$ symmetry
- Broken by coupling of phonons to baryons ($-\lambda\,\theta\rho_b$)
- Heuristically
  - Chemical potential: $\theta = \mu \cdot t$
  - How long can you ignore time-dependence from coupling?
- Superfluid in equilibrium with chemical potential can exist only on timescales shorter than

$$t_Q \sim \frac{1}{\lambda m}\frac{M_{DM}}{M_b} \sim 10^8\,\mathrm{yr}\cdot\frac{M_{DM}}{M_b}$$

- Not much larger than galactic timescales
- Local version of this estimate is even more constraining.

The root cause

- One field has two jobs:
  - θ mediates a MOND force
  - θ carries the superfluid

$\rightarrow$ These are in tension with each other
  - E.g. to fix the "MOND limit problem" $\rightarrow$ small λm
  - But: Significant superfluid density ρ_{SF} $\rightarrow$ large λm

A solution: two-field SFDM

- Solution: Split jobs between θ_+ (carries the MOND force) and θ_- (carries the superfluid).

$$\mathcal{L}_{\rm standard} = f(K - m^2) - \lambda\,\theta\,\rho_b\,,$$

$$\downarrow$$

$$\mathcal{L}_{\rm two-field} = \mathcal{L}_- + f(K_+ + K_- - m^2) - \lambda\,\theta_+\,\rho_b\,,$$

$\mathcal{L}_-$ = standard superfluid Lagrangian with phase θ_-
$f(K) \sim K^{3/2}$ as in standard SFDM, contains both θ_+ and θ_-.

✓ Long-lived equilibrium with $\dot{\theta}_- = m + \mu_{\rm nr}$

✓ Proper MOND limit, i.e. $2m\hat{\mu} \ll (\vec{\nabla}\theta_+)^2$

✓ Roughly similar SF profile as standard SFDM

? Transition from superfluid core to NFW halo (also unclear in standard SFDM)

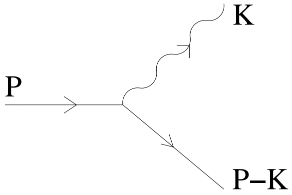
Another test:
Cherenkov radiation from stars

[TM 2021, not yet peer-reviewed]

Cherenkov radiation

Electromagnetic Cherenkov radiation

- Matter can lose energy if $V > c_s$
- Requirements:
  - Mode coupled to matter
  - Mode has $\omega = c_s k$ with $c_s < 1$



[Moore, Nelson 2009]

In Modified Gravity models

✓ Modified gravity mode coupled to matter

✓ Often with $c_s \approx 1$ but $c_s < 1$

$\rightarrow$ Cherenkov radiation possible, but only for relativistic objects

$\rightarrow$ e.g. cosmic rays with $V > c_s$ lose energy, radiate away modified gravity mode $\rightarrow$ Contraints

Cherenkov radiation in hybrid models

Hybrid models
(with common origin for galactic and cosmological phenomena)

- For MOND in galaxies $\rightarrow$ Mode that is coupled to matter
- For CDM in cosmology $\rightarrow$ Perfect fluid with $c_s \ll 1$
- With common origin: Both are related. So:
  - ✓ Mode that is coupled to matter
  - ✓ This mode propagates with $c_s \neq 1$, even $c_s \ll 1$

$\rightarrow$ Cherenkov radiation possible even for **non-relativistic** objects

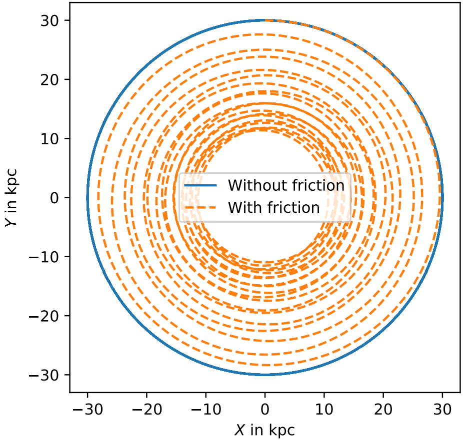
$\rightarrow$ e.g. stars with $V > c_s$ lose energy $\rightarrow$ Constraints

Example: SFDM

- Phonons are coupled to matter + propagate with $c_s \ll 1$
- Stars with $V > c_s$ lose energy by radiating away phonons

Cherenkov radiation from stars: Effects

For $V > \mathcal{O}(c_s)$: Energy loss timescale $\tau_E \equiv \frac{E}{|\dot{E}|} \sim \frac{10^8\,\mathrm{yr}}{g_m^2} \left(\frac{V}{c_s}\right)^2$



Cherenkov radiation from stars: Calculation

Background galaxy

$\downarrow$

Perturbations (δ_b: the star, δ: the radiation mode e.g. phonons)

$$\mathcal{L} = \frac{1}{2}\frac{1}{\bar{c}^2}(\partial_t\delta)^2 - \frac{1}{2}\left((\vec{\nabla}\delta)^2 + (\hat{a}\vec{\nabla}\delta)^2\right) - \frac{g_m}{\sqrt{2}M_{\mathrm{Pl}}}\delta\,\delta_b\,,$$

$\downarrow$

$$\dot{E} = -\int^{k_{\mathrm{max}}} \omega d\Gamma$$

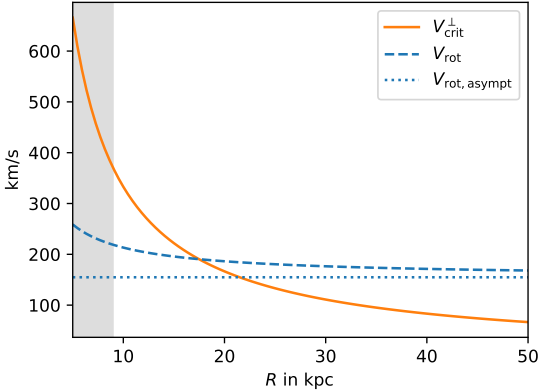
Cuts: Perturbations stay small, stay in MOND regime

$\rightarrow$ Calculated $|\dot{E}|$ is lower bound

$\rightarrow$ acts like a friction force

Standard SFDM constraints

For galaxy in MOND limit: $c_s \propto a_\theta / a_0 \propto 1/R$



Ruled out unless either:

- ? $V < c_s$ (Cherenkov radiation kinematically forbidden)
- ? $\tau_E > \tau_{\min}$ (Cherenkov radiation allowed, but lose little energy)

Standard SFDM constraints

- For standard SFDM at fixed R (because $g_m = \mathcal{O}(1)$):

$$\tau_E \propto 1/c_s^2$$

- Ruled out unless either:
  - ? c_s large ($V_{\rm crit}$ is large)
  - ? c_s is small (τ_E is large)

$\rightarrow$ Rules out interval of c_s

$\rightarrow$ Rules out interval of $\sqrt{\alpha}/m$
($c_s \propto \sqrt{\alpha}/m$ with $\alpha = a_0/(\lambda M_{\rm Pl})$)

- Above: Neglected β-dependent prefactors

$\rightarrow$ Rule out interval of $\sqrt{\alpha}/m$ for fixed values of β

Standard SFDM constraints

- Use observed Milky Way rotation curve
- Require either: Energy loss timescale $\gtrsim 10^{10}\,\mathrm{yr}$
  or: no Cherenkov radiation
- Rule out $\sqrt{\alpha}/m \in (q_l, q_h) \cdot \mathrm{eV}^{-1}$ for fixed β

R kpc	V km/s	(q_l, q_h) for $\beta = 3/2$	(q_l, q_h) for $\beta = 2$	(q_l, q_h) for $\beta = 3$
15.2	220^{+1}_{-1}	$(0.25, 1.56)$	$(0.34, 2.19)$	$(0.51, 3.34)$
20.3	203^{+3}_{-3}	$(0.35, 1.92)$	$(0.46, 2.70)$	$(0.69, 4.11)$
24.8	202^{+6}_{-6}	$(0.47, 2.34)$	$(0.62, 3.29)$	$(0.93, 5.01)$

- E.g. for $\beta = 2$ rule out (standard: $\beta = 2$, $\sqrt{\alpha}/m = 2.4\,\mathrm{eV}^{-1}$)

$$\boxed{0.34\,\mathrm{eV}^{-1} \lesssim \sqrt{\alpha}/m \lesssim 3.29\,\mathrm{eV}^{-1}}$$

$\rightarrow$ MOND limit in MW with these parameters ruled out

Other models?

- All hybrid models have to deal with this type of constraint, if cosmological and galactic phenomena share common origin
- No common origin e.g. in νHDM
- Otherwise: Two mechanisms to avoid by having $\tau_E \gg 10^{10}\,\mathrm{yr}$

Weaken link between galactic and cosmological phenomena

- Two-field SFDM does this
- θ_+: Directly coupled to matter, but relativistic sound speed
- θ_-: Non-relativistic sound speed, but coupled only indirectly

Suppress coupling in dynamical situations

- Recent model by Skordis & Złośnik does this
- Mode ϕ is coupled directly to matter and has (potentially) non-relativistic sound speed
- But: Coupling is suppressed by powers of $1/\omega$ in dynamical situations ($\omega \neq 0$)

Summary

- Hybrid MOND dark matter models are phenomenologically well-motivated
- Can fit strong lensing and Milky Way rotation curve
- Standard SFDM: Theoretical issues due to double role of phonon field
- Requires theoretical developments, e.g. two-field SFDM
- Hybrid models with common origin for MOND/CDM $\rightarrow$ Cherenkov radiation from stars
- Gives new type of constraint for such models
- Rules out parameter space for standard SFDM.
- Special mechanisms can avoid constraints (e.g. two-field SFDM and recent model by Skordis & Złośnik)