Einstein-Cartan Gravity in Particle Physics and Cosmology

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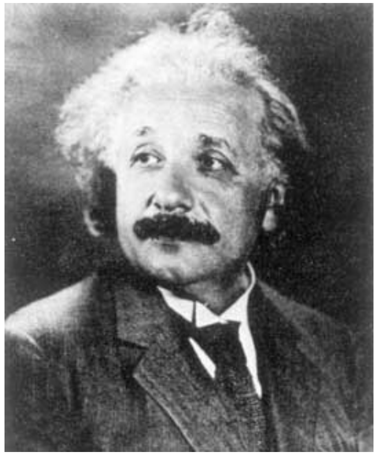
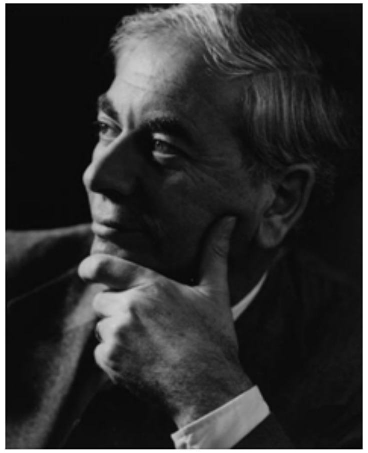
22/23 November 2016

Outline

1. Einstein-Cartan-Sciama-Kibble gravity
2. Spin-torsion coupling of Dirac fields
3. Matter-antimatter asymmetry from torsion
4. Nonsingular fermions from torsion
5. Spin fluids
6. Big bounce and inflation from torsion
7. Cosmological constant from torsion

Gravity with torsion

Einstein-Cartan-Sciama-Kibble

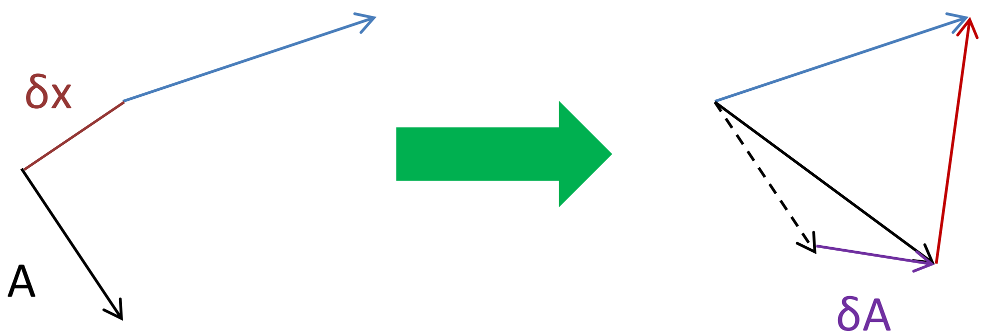
theory of gravity

What is torsion?

- Tensors – behave under coordinate transformations like products of differentials and gradients. Special case: vectors.

- Differentiation of vectors in curved spacetime requires subtracting two infinitesimal vectors at two points that have different transformation properties.

- Parallel transport allows to bring one vector to the origin of the other one, so that their difference would make sense.



$$\delta A^i = -\Gamma^i_{jk} A^j \delta x^k$$

↓

Affine connection

What is torsion?

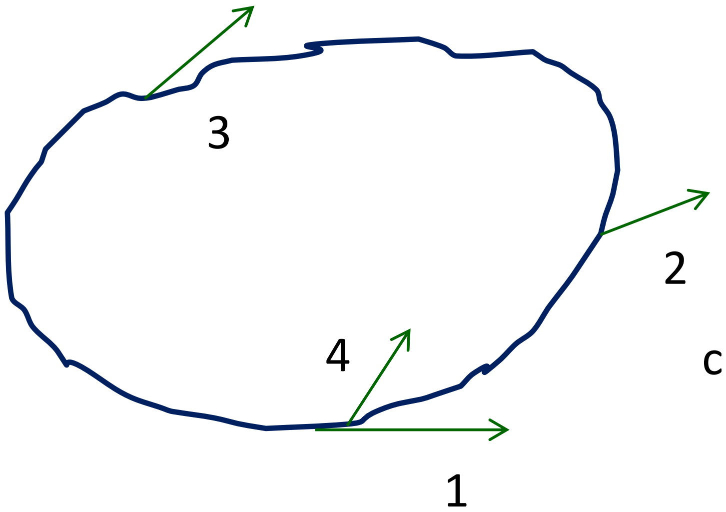
- Curved spacetime requires geometrical structure: affine connection $\Gamma^{\rho}{}_{\mu\nu}$

- Covariant derivative

$$\nabla_{\nu} V^{\mu} = \partial_{\nu} V^{\mu} + \Gamma^{\mu}{}_{\rho\nu} V^{\rho}$$

- Curvature tensor

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\sigma\nu} - \partial_{\nu}\Gamma^{\rho}{}_{\sigma\mu} + \Gamma^{\rho}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\sigma\nu} - \Gamma^{\rho}{}_{\lambda\nu}\Gamma^{\lambda}{}_{\sigma\mu}$$



Measures the change of a vector parallel-transported along a closed curve:

change = curvature X area X vector

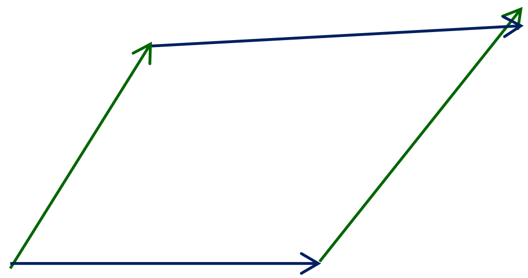
What is torsion?

- **Torsion tensor** – antisymmetric part of affine connection

$$S^k{}_{ij} = \Gamma^{\;k}_{[i\,j]}$$

- Contortion tensor

$$C^i{}_{jk} = S^i{}_{jk} + S_{jk}{}^i + S_{kj}{}^i$$



Measures noncommutativity of parallel transports

GR – affine connection restricted to be symmetric in lower indices

ECSK – no constraint on connection: more natural

Theories of spacetime

Special Relativity – flat spacetime (no curvature)
Dynamical variables: matter fields

$$g_{ik} = \eta_{ik}$$

↓

General Relativity – (curvature, no torsion)
Dynamical variables: matter fields + metric tensor g_{ik}

$$S^k{}_{ij} = 0$$

↓

ECSK Gravity (simplest theory with curvature & **torsion**)
Dynamical variables: matter fields + metric g_{ik} + torsion $S^k{}_{ij}$

More degrees of freedom

ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Riemann-Cartan spacetime – metricity $\mathrm{r}_{½}\mathrm{g}_{10} = 0$

→ connection $¡^{½}{}_{10} = \{^{½}{}_{10}\} + C^{½}{}_{10}$

Christoffel symbols ↗ ↖ contortion tensor

- Lagrangian density for matter $\mathfrak{L}$

Metrical energy-momentum tensor

$$T_{ik} = \frac{2}{\sqrt{-g}} \frac{\delta \mathfrak{L}}{\delta g^{ik}}$$

Spin tensor

$$s^{ijk} = \frac{2}{\sqrt{-g}} \frac{\delta \mathfrak{L}}{\delta C_{ijk}}$$

Total Lagrangian density $-\frac{R\sqrt{-g}}{2\kappa} + \mathfrak{L}$ (like in GR)

arXiv.org > gr-qc > arXiv:0911.0334

ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Curvature tensor = Riemann tensor
  + tensor quadratic in torsion + total derivative

- Stationarity of action under $\pm g^{10}$ → Einstein equations

$$R^{\{\}}{}_{10} - R^{\{\}} g_{10}/2 = k(T_{10} + U_{10})$$

$$U_{10} = [C^{½}{}_{1½}C^{¾}{}_{0¾} - C^{½}{}_{1¾}C^{¾}{}_{0½} - (C^{½¾}{}_{½}C^{¿}{}_{¾¿} - C^{¾½¿}C_{¿½¾})g_{10}/2]/k$$

- Stationarity of action under $\pm C^{10}{}_{½}$ → Cartan equations

$$S^{½}{}_{10} - S_1 \pm^{½}{}_{0} + S_0 \pm^{½}{}_{1} = -k s_{10}{}^{½}/2$$

Same coupling constant k

$$S_1 = S^{o}{}_{10}$$

- Cartan equations are algebraic and linear: **torsion α spin density**
- Contributions to energy-momentum from spin are quadratic

ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Field equations with full Ricci tensor can be written as

$R_{10} - Rg_{10}/2 = k\pounds_{01}$

Tetrad energy-momentum tensor

- Belinfante-Rosenfeld relation

$\pounds_{10} = T_{10} + r^{¤}{}_{½}(s_{10}{}^{½} + s^{½}{}_{01} + s^{½}{}_{10})/2$

$r^{¤}{}_{½} = r_{½} - 2S_{½}$

- Conservation law for spin

$r^{¤}{}_{½}s_{10}{}^{½} = (\pounds_{10} - \pounds_{01})$

- Cyclic identities

$R^{¾}{}_{10½} = -2r_{1}S^{¾}{}_{0½} + 4S^{¾}{}_{¿1}S^{¿}{}_{0½}$

(^{1}, º, ½ cyclically permutated)

ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

- Bianchi identities (¹, º, ½ cyclically permutated)

$$\mathrm{r}_{1}\mathrm{R}^{¾}{}_{¿\mathrm{o}1½} = 2\mathrm{R}^{¾}{}_{¿¼1}\mathrm{S}^{¼}{}_{\mathrm{o}1½}$$

- Conservation law for energy and momentum

$$\mathrm{D}_{\mathrm{o}}£^{1\mathrm{o}} = \mathrm{C}_{\mathrm{o}1½}{}^{1}£^{\mathrm{o}1½} + \mathrm{s}_{\mathrm{o}1½¾}\mathrm{R}^{\mathrm{o}1½¾1}/2 \qquad \mathrm{D}_{\mathrm{o}}=\mathrm{r}^{\{\}}{}_{\mathrm{o}}$$

Equations of motion of particles

F. W. Hehl, P. von der Heyde, G. D. Kerlick & J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976)
E. A. Lord, *Tensors, Relativity and Cosmology* (McGraw-Hill, 1976) – @ Indian Institute of Science, Bangalore, India
NJP, arXiv:0911.0334

ECSK gravity

- No spinors -> torsion vanishes -> ECSK reduces to GR

- Torsion significant when U_{10} » T_{10} (at Cartan density)

For fermionic matter (quarks and leptons)

$$\rho_C = \frac{m_n^2 c^4}{G\hbar^2}$$

½ > 10^{45} kg m^{-3}

Nuclear matter in neutron stars

½ » 10^{17} kg m^{-3}

Gravitational effects of torsion negligible even for neutron stars

Torsion significant only in very early Universe and in black holes, and for fermions at very small scales

Spin-torsion coupling of spinors

- Dirac matrices $\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab}I$

$$g_{ik} = e_i^a e_k^b \eta_{ab}$$

Tetrad

- Spinor representation of Lorentz group

$$\gamma^a = \Lambda^a{}_b L\gamma^b L^{-1}$$

- Spinors

$$\tilde{\psi} = L\psi \qquad \tilde{\bar{\psi}} = \bar{\psi}L^{-1}$$

$$\omega^i{}_{ak} = e^i_{a;k}$$

Spin connection

$$\omega^a{}_{bi} = -\omega_b{}^a{}_i$$

Spinor connection

- Covariant derivative of spinor $\psi_{;i} = \psi_{,i} - \Gamma_i\psi$

$$V^a{}_{|i} = V^a{}_{,i} + \omega^a{}_{bi}V^b$$

$$\gamma^a{}_{|i} = \omega^a{}_{bi}\gamma^b - [\Gamma_i, \gamma^a]$$

$$e^i_{a|k} = e^i_{a,k} + \Gamma^{\,i}_{j\,k}e^j_a - \omega^b{}_{ak}e^i_b = 0$$

Metricity -> $\gamma^a{}_{|i} = 0$ -> $\Gamma_i = -\frac{1}{4}\omega_{abi}\gamma^a\gamma^b$

Fock-Ivanenko coefficients (1929)

Spin-torsion coupling of spinors

- Dirac Lagrangian density

$$\mathfrak{L} = \frac{i\sqrt{-g}}{2}(\bar{\psi}\gamma^i\psi_{;i} - \bar{\psi}_{;i}\gamma^i\psi) - m\sqrt{-g}\bar{\psi}\psi$$

; – covariant derivative with affine connection

: – with Christoffel symbols

- Spin density

$$s^{ijk} = \frac{2}{\sqrt{-g}}\frac{\delta\mathfrak{L}}{\delta C_{ijk}}$$

Variation of C ⇔ variation of ω

Totally antisymmetric

$$s^{ijk} = \frac{i}{2}\bar{\psi}\gamma^{[i}\gamma^j\gamma^{k]}\psi = s^{[ijk]} = -e^{ijkl}s_l$$

$$s^i = \frac{1}{2}\bar{\psi}\gamma^i\gamma^5\psi$$ Dirac spin pseudovector

Cartan equations

$$C_{ijk} = \frac{\kappa}{4}\epsilon_{ijkl}\bar{\psi}\gamma^l\gamma^5\psi$$

Dirac equation

$$i\gamma^k(\psi_{:k} + \frac{1}{4}C_{ijk}\gamma^i\gamma^j\psi) = m\psi$$

$$i\gamma^k\psi_{:k} + \frac{3\kappa}{8}(\bar{\psi}\gamma_k\gamma^5\psi)\gamma^k\gamma^5\psi = m\psi$$

F. W. Hehl & B. K. Datta, J. Math. Phys. **12**, 1334 (1971) – @ Bose Institute, India

Matter-antimatter asymmetry

NJP, Phys. Rev. D **83**, 084033 (2011)

- Hehl-Datta equation

$$ie^{\mu}_{a}\gamma^{a}\psi_{:\mu} + qe^{\mu}_{a}A_{\mu}\gamma^{a}\psi = m\psi - \underline{\frac{3\kappa}{8}(\bar{\psi}\gamma^{5}\gamma_{a}\psi)\gamma^{5}\gamma^{a}\psi}.$$

$$\bar{\psi} = \psi^{\dagger}\gamma^{0}$$

Adjoint spinor

- Charge conjugate

$$\psi^{c} = -i\gamma^{2}\psi^{*}$$

Satisfies Hehl-Datta equation with opposite charge and different sign for the cubic term

$$ie^{\mu}_{a}\gamma^{a}\psi^{c}_{:\mu} - qe^{\mu}_{a}A_{\mu}\gamma^{a}\psi^{c} = m\psi^{c} + \underline{\frac{3\kappa}{8}(\bar{\psi}^{c}\gamma^{5}\gamma_{a}\psi^{c})\gamma^{5}\gamma^{a}\psi^{c}}.$$

Energy levels (effective masses)

Fermions $\omega = m + \alpha\kappa N$

Antifermions $\omega = m - \alpha\kappa N$

↑ Inverse normalization for spinor wave function

HD asymmetry significant when torsion is

-> baryogenesis -> dark matter?

Matter-antimatter asymmetry

PHYSICAL REVIEW D **83,** 084033 (2011)

Matter-antimatter asymmetry and dark matter from torsion

Nikodem J. Popławski*

We propose a simple scenario which explains the observed matter-antimatter imbalance and the origin of dark matter in the Universe. We use the Einstein-Cartan-Sciama-Kibble theory of gravity which naturally extends general relativity to include the intrinsic spin of matter. Spacetime torsion produced by spin generates, in the classical Dirac equation, the Hehl-Datta term which is cubic in spinor fields. We show that under a charge-conjugation transformation this term changes sign relative to the mass term. A classical Dirac spinor and its charge conjugate therefore satisfy different field equations. Fermions in the presence of torsion have higher energy levels than antifermions, which leads to their decay asymmetry. Such a difference is significant only at extremely high densities that existed in the very early Universe. We propose that this difference caused a mechanism, according to which heavy fermions existing in such a Universe and carrying the baryon number decayed mostly to normal matter, whereas their antiparticles decayed mostly to hidden antimatter which forms dark matter. The conserved total baryon number of the Universe remained zero.

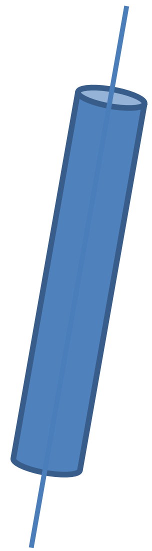
Multipole expansion

- Papapetrou (1951) – multipole expansion -> equations of motion

Matter in a small region in space with coordinates $x^{¹}(s)$

Motion of an extended body – world tube

Motion of the body as a whole – wordline $X^{¹}(s)$



- $\pm x^{®} = x^{®} - X^{®}$ $\pm x^{0} = 0$

  $u^{¹} = dX^{¹}/ds$ ® - spatial coordinates

Four-velocity

$M^{¹0½} = -u^{0}\, s \pm x^{¹}\, £^{0½} (-g)^{1/2}\, dV$

$N^{¹0½} = u^{0}\, s s^{¹0½} (-g)^{1/2}\, dV$

- Dimensions of the body small -> neglect higher-order (in $\pm x^{¹}$) integrals and omit surface integrals

Nonsingular fermions

$$M^{0jk} = 0$$

s -> Σ

$$\int \Sigma^{ij0}{}_{,0}\, dV - \int \Gamma_l{}^i{}_k \Sigma^{jlk}\, dV + \int \Gamma_l{}^j{}_k \Sigma^{ilk}\, dV$$
$$- 2\int \Theta^{[ij]}\, dV = 0.$$

$$(x^l \Sigma^{ijk})_{,k} = \Sigma^{ijl} + x^l \Gamma^i{}_l{}^k \Sigma^{jlk} - x^l \Gamma_l{}^j{}_k \Sigma^{ilk} + 2x^l \Theta^{[ij]}$$

$$\int (x^l \Sigma^{ij0})_{,0}\, dV = \int \Sigma^{ijl}\, dV + \int x^l \Gamma_m{}^i{}_k \Sigma^{jmk}\, dV$$
$$- \int x^l \Gamma_m{}^j{}_k \Sigma^{imk}\, dV + 2\int x^l \Theta^{[ij]}\, dV$$

$$\frac{u^l}{u^0} \int \Sigma^{ij0}\, dV + X^l \int \Sigma^{ij0}{}_{,0}\, dV$$
$$= \int \Sigma^{ijl}\, dV + 2\int \delta x^l \Theta^{[ij]}\, dV$$
$$+ X^l \left(\int \Gamma_m{}^i{}_k \Sigma^{jmk}\, dV - \int \Gamma_m{}^j{}_k \Sigma^{imk}\, dV + 2\int \Theta^{[ij]}\, dV \right),$$

K. Nomura, T. Shirafuji & K. Hayashi, Prog. Theor. Phys. **86**, 1239 (1991)

Nonsingular fermions

$$\frac{u^l}{u^0}\int \Sigma^{ij0}\,dV = \int \Sigma^{ijl}\,dV + 2\int \delta x^l \Theta^{[ij]}\,dV$$

$$M^{l[ij]} = -\frac{1}{2}\left(\frac{u^l}{u^0}N^{ij0} - N^{ijl}\right)$$

For Dirac fields: $N^{ijk} = N^{[ijk]}$ $\quad M^{\alpha ij} \propto \int \delta x^\alpha u^{ij} \delta(\mathbf{r})\,dV$

$M^{\alpha ij} = 0$ $\quad N^{ijl} = \frac{u^l}{u^0}N^{ij0}$ $\quad N^{il0} = -\frac{u^l}{u^0}N^{i00} = 0$

$N^{ijk} = 0$ $\quad \Sigma^{ijk} = 0$ $\quad \psi = 0$

Single-pole approximation of Dirac field contradicts field equations. Dirac fields cannot represent point particles (also 1D and 2D configurations). Fermions must be 3D.

NJP, Phys. Lett. B **690**, 73 (2010)

Nonsingular fermions

For Dirac fields:

The metric energy–momentum tensor for the Lagrangian density is on the order of $mc^2|\psi|^2$ (in the rest frame of the particle), the spin density is on the order of $\hbar c|\psi|^2$, and the wave function $\psi \sim d^{-3/2}$. Thus this size is on the order of the Cartan radius r_C

$$\frac{m}{r_C^3} \sim \frac{G}{c^4}\left(\frac{\hbar}{r_C^3}\right)^2.$$

For an electron, $r_{Ce} \sim 10^{-27}$ m,

NJP, Phys. Lett. B **690**, 73 (2010)

Nonsingular fermions

Physics Letters B 690 (2010) 73–77

Nonsingular Dirac particles in spacetime with torsion

Nikodem J. Popławski

We use the Papapetrou method of multipole expansion to show that a Dirac field in the Einstein–Cartan–Kibble–Sciama (ECKS) theory of gravity cannot form singular configurations concentrated on one- or two-dimensional surfaces in spacetime. Instead, such a field describes a nonsingular particle whose spatial dimension is at least on the order of its Cartan radius. In particular, torsion modifies Burinskii's model of the Dirac electron as a Kerr–Newman singular ring of the Compton size, by replacing the ring with a toroidal structure with the outer radius of the Compton size and the inner radius of the Cartan size. We conjecture that torsion produced by spin prevents the formation of singularities from matter composed of quarks and leptons. We expect that the Cartan radius of an electron, $\sim 10^{-27}$ m, introduces an effective ultraviolet cutoff in quantum field theory for fermions in the ECKS spacetime. We also estimate a maximum density of matter to be on the order of the corresponding Cartan density, $\sim 10^{51}$ kg m^{-3}, which gives a lower limit for black-hole masses $\sim 10^{16}$ kg. This limit corresponds to energy $\sim 10^{43}$ GeV which is 39 orders of magnitude larger than the maximum beam energy currently available at the LHC. Thus, if torsion exists and the ECKS theory of gravity is correct, the LHC cannot produce micro black holes.

Spin fluids

- Conservation law for spin ->

$$M^{½10} - M^{½01} = N^{10½} - N^{10}{}^{0}\, u^{½}/u^{0}$$

- Average fermionic matter as a continuum (fluid)

Neglect $M^{½10}$ -> $s^{10½} = s^{10} u^{½}$ $\qquad s^{10} u_{o} = 0$

Macroscopic spin tensor of a **spin fluid**

- Conservation law for energy and momentum ->

$$£^{10} = c_{|}^{|1} u^{o} - p(g^{10} - u^{1} u^{o}) \qquad {}^{2} = c_{|1}^{|} u^{1} \qquad s^{2} = s^{10} s_{10}/2$$

Four-momentum density — Pressure — Energy density

J. Weyssenhoff & A. Raabe, Acta Phys. Pol. **9**, 7 (1947)

Spin fluids

- Dynamical energy-momentum tensor for a spin fluid

$$T^{ij} + U^{ij} = \left(\epsilon - \frac{1}{4}\kappa s^2\right) u^i u^j - \left(p - \frac{1}{4}\kappa s^2\right) (g^{ij} - u^i u^j)$$

Energy density **Pressure**

$$-(\delta^l_k + u_k u^l) D_l (s^{k(i} u^{j)})$$

for random spin orientation

F. W. Hehl, P. von der Heyde & G. D. Kerlick, Phys. Rev. D **10**, 1066 (1974)

- Spin fluid of fermions with no spin polarization

$$s^2 = \frac{1}{8}(\hbar c n)^2$$

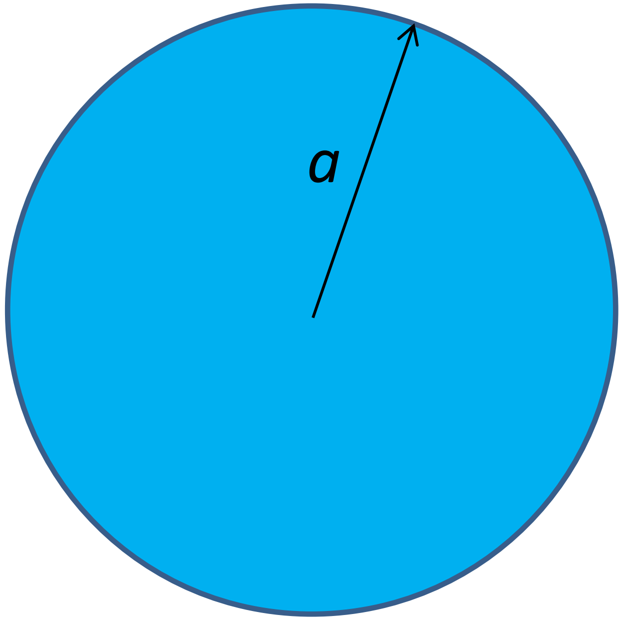
I. S. Nurgaliev & W. N. Ponomariev, Phys. Lett. B **130**, 378 (1983)

Bounce cosmology with torsion

Closed, homogeneous & isotropic Universe: 3D surface of 4D sphere

Friedman-Lemaitre-Robertson-Walker metric (k = 1)

$$ds^2 = c^2dt^2 - \frac{a^2(t)}{(1+kr^2/4)^2}(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2)$$



Friedman equations for scale factor *a(t)*

$$\dot{a}^2 + 1 = \frac{1}{3}\kappa\Big(\epsilon - \frac{1}{4}\kappa s^2\Big)a^2,$$

$$\dot{a}^2 + 2a\ddot{a} + 1 = -\kappa\Big(p - \frac{1}{4}\kappa s^2\Big)a^2$$

Conservation law

$$\frac{d}{dt}((\epsilon - \kappa s^2/4)a^3) + (p - \kappa s^2/4)\frac{d}{dt}(a^3) = 0$$

B. Kuchowicz, Gen. Relativ. Gravit. **9**, 511 (1978)
M. Gasperini, Phys. Rev. Lett. **56**, 2873 (1986)
NJP, Phys. Lett. B **694**, 181 (2010)

Spin-torsion coupling generates negative energy (gravitational repulsion)

Bounce cosmology with torsion

Physics Letters B 694 (2010) 181–185

Cosmology with torsion: An alternative to cosmic inflation

Nikodem J. Popławski

We propose a simple scenario which explains why our Universe appears spatially flat, homogeneous and isotropic. We use the Einstein–Cartan–Kibble–Sciama (ECKS) theory of gravity which naturally extends general relativity to include the spin of matter. The torsion of spacetime generates gravitational repulsion in the early Universe filled with quarks and leptons, preventing the cosmological singularity: the Universe expands from a state of minimum but finite radius. We show that the dynamics of the closed Universe immediately after this state naturally solves the flatness and horizon problems in cosmology because of an extremely small and negative torsion density parameter, $\Omega_S \approx -10^{-69}$. Thus the ECKS gravity provides a compelling alternative to speculative mechanisms of standard cosmic inflation. This scenario also suggests that the contraction of our Universe preceding the bounce at the minimum radius may correspond to the dynamics of matter inside a collapsing black hole existing in another universe, which could explain the origin of the Big Bang.

Torsion and particle production

For relativistic matter in thermal equilibrium, Friedman equations can be written in terms of temperature.

$$\frac{\dot{a}^2}{c^2} + k = \frac{1}{3}\kappa\tilde{\epsilon}a^2 = \frac{1}{3}\kappa(h_\star T^4 - \alpha h_{\rm nf}^2 T^6)a^2, \qquad \alpha = \kappa(\hbar c)^2/32$$

$$\frac{\dot{a}}{a} + \frac{\dot{T}}{T} = \frac{cK}{3h_{n1}T^3}, \qquad K = \beta(\kappa\tilde{\epsilon})^2,$$

2nd Friedman equation is rewritten as 1st law of thermodynamics for constant entropy. Parker-Starobinskii-Zel'dovich particle production rate *K*, proportional to the square of curvature, produces entropy in the Universe. No reheating needed.

NP, Astrophys. J. **832**, 96 (2016)

Generating inflation with only 1 parameter

Near a bounce:

$$\frac{\dot{a}}{a}\left[1-\frac{3\beta}{c^3 h_{n1} T^3}\left(\frac{\dot{a}}{a}\right)^3\right]=-\frac{\dot{T}}{T}$$

To avoid eternal inflation: $\beta < \beta_{\rm cr} = \frac{\sqrt{6}}{32}\frac{h_{n1}h_{n\rm f}^3(\hbar c)^3}{h_\star^3} \approx \frac{1}{929}$

During an expansion phase, near critical value of particle production coefficient β:

$$\dot{T}\approx 0 \qquad \frac{\dot{a}}{a}\approx\frac{c\beta(\kappa\tilde{\epsilon})^2}{3h_{n1}T^3}\approx c\left(\frac{1}{3}\kappa\tilde{\epsilon}\right)^{1/2} \qquad \tilde{\epsilon}\approx\frac{h_\star^3}{8\alpha^2 h_{n\rm f}^4}$$

Exponential expansion lasts about $\tau = \frac{\alpha h_{n\rm f}^2}{c}\left(\frac{3}{\kappa h_\star^3}\right)^{1/2}$ then *T* decreases.

Torsion becomes weak and radiation dominated era begins.
No hypothetical fields needed.

Universe in a Black Hole in Einstein–Cartan Gravity

Nikodem Popławski

Astrophys. J. **832**, 96 (2016)

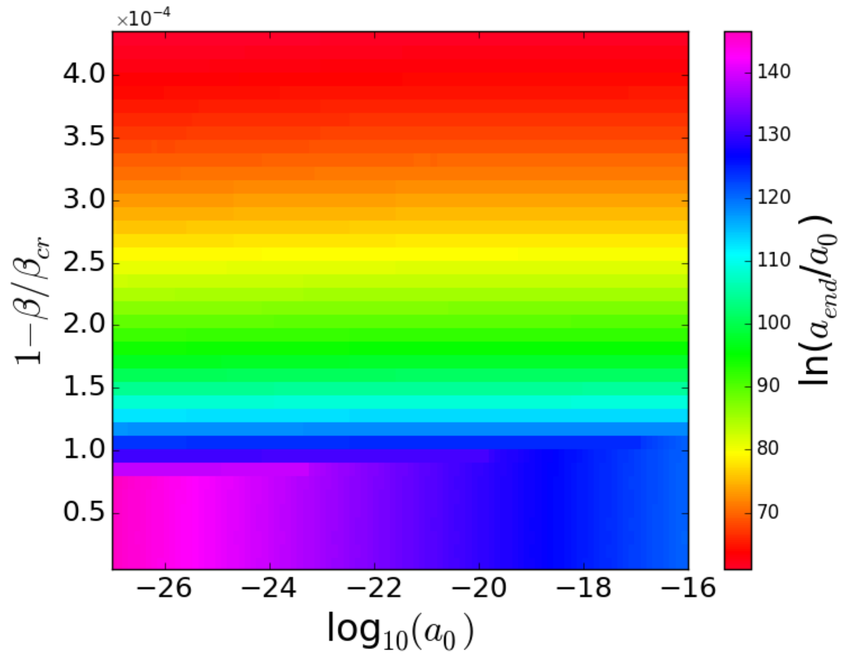
Abstract

The conservation law for the angular momentum in curved spacetime, consistent with relativistic quantum mechanics, requires that the antisymmetric part of the affine connection (torsion tensor) is a variable in the principle of least action. The coupling between the spin of elementary particles and torsion in the Einstein–Cartan theory of gravity generates gravitational repulsion at extremely high densities in fermionic matter, approximated as a spin fluid, and thus avoids the formation of singularities in black holes. The collapsing matter in a black hole should therefore bounce at a finite density and then expand into a new region of space on the other side of the event horizon, which may be regarded as a nonsingular, closed universe. We show that quantum particle production caused by an extremely high curvature near a bounce can create enormous amounts of matter, produce entropy, and generate a finite period of exponential expansion (inflation) of this universe. This scenario can thus explain inflation without a scalar field and reheating. We show that, depending on the particle production rate, such a universe may undergo several nonsingular bounces until it has enough matter to reach a size at which the cosmological constant starts cosmic acceleration. The last bounce can be regarded as the big bang of this universe.

- The temperature at a bounce depends on the number of elementary particles and the Planck temperature.

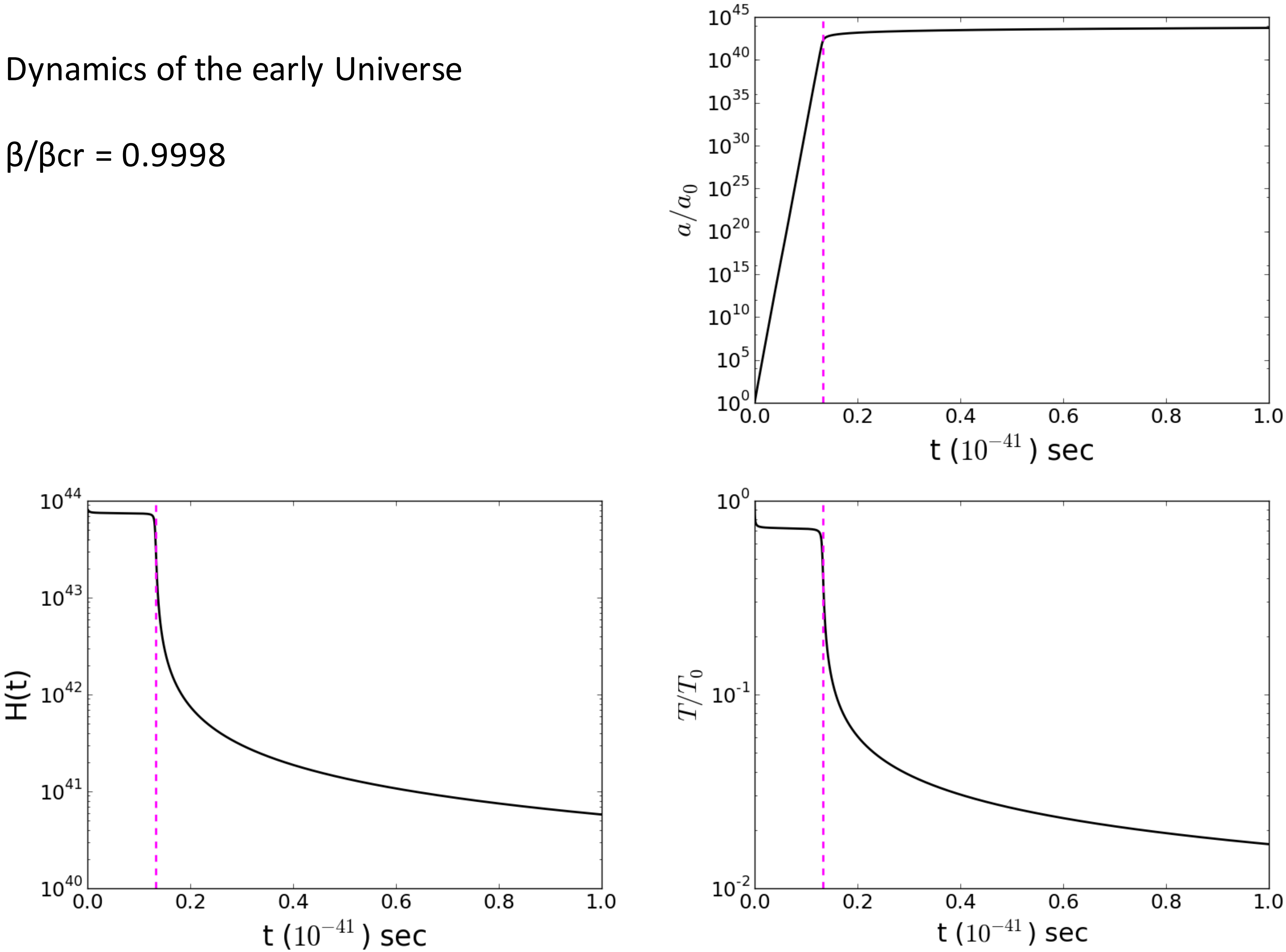
- Numerical integration of the equations shows that the numbers of bounces and e-folds depend on the particle production coefficient but are not too sensitive to the initial scale factor.

β/β_{cr}	Number of bounces
0.996	1
0.984	2
0.965	3
0.914	5
0.757	10

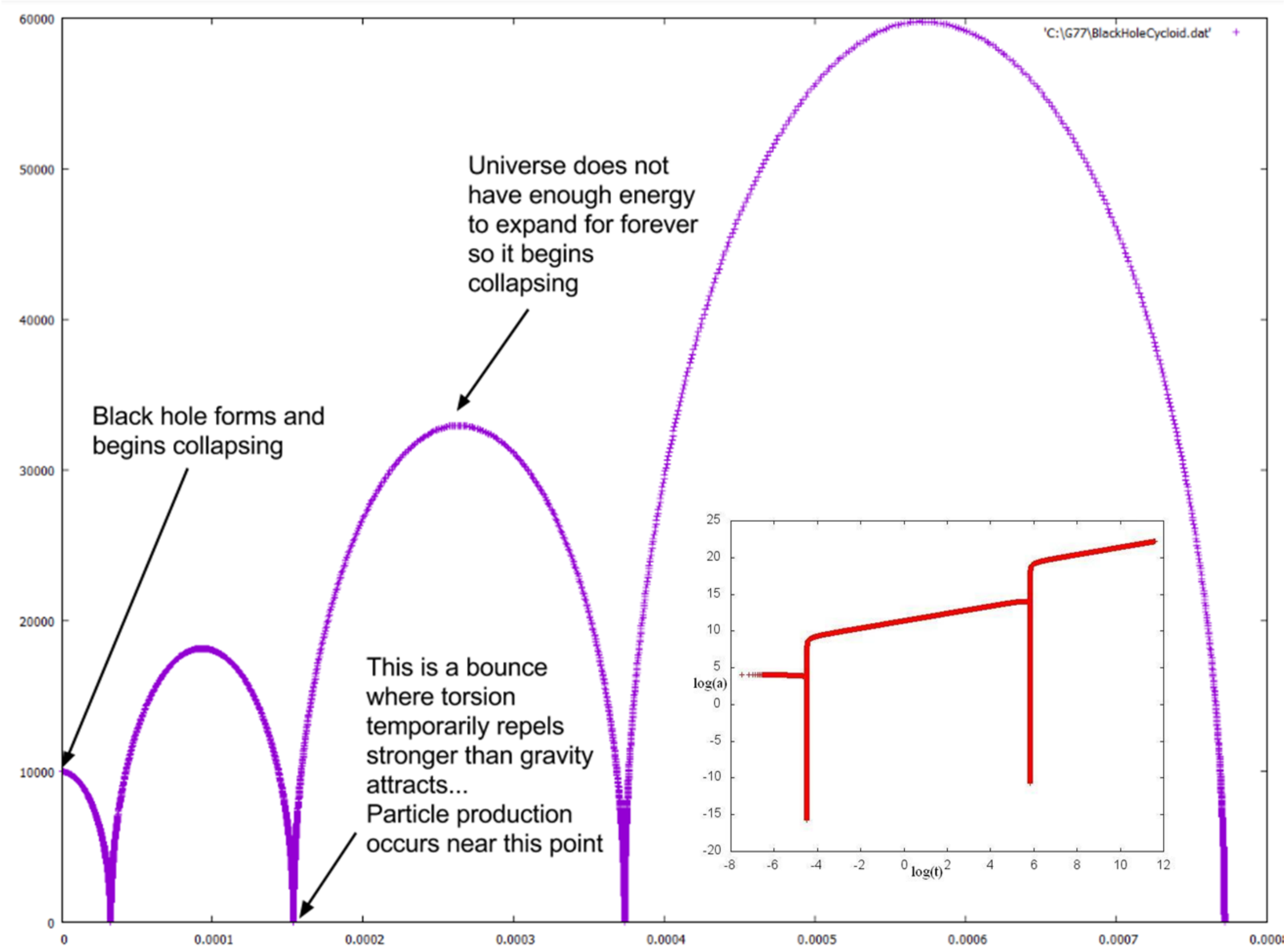


Dynamics of the early Universe

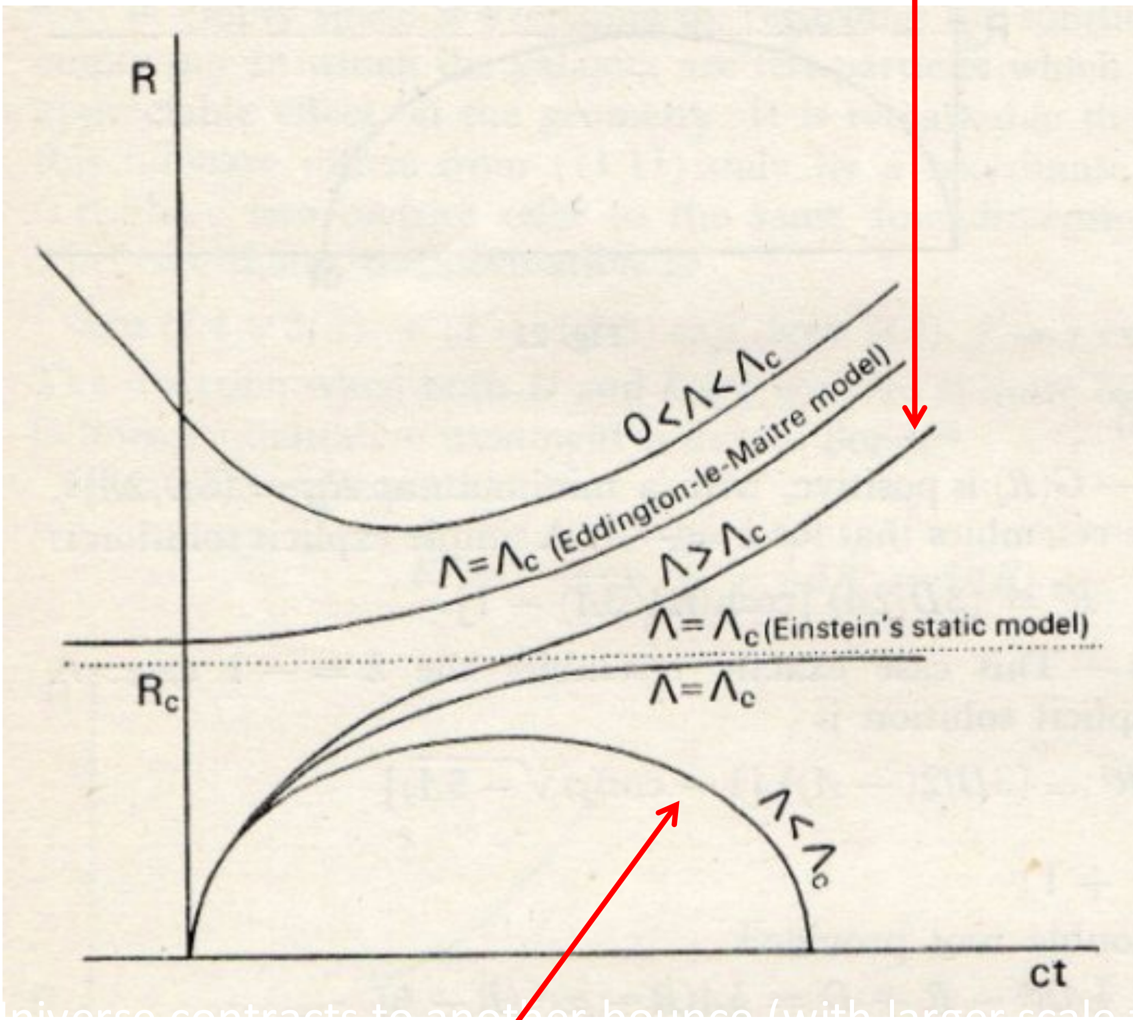
β/βcr = 0.9998



S. Desai & NJP, Phys. Lett. B **755**, 183 (2016)


'C:\G77\BlackHoleCycloid.dat'
Universe does not have enough energy to expand for forever so it begins collapsing
Black hole forms and begins collapsing
This is a bounce where torsion temporarily repels stronger than gravity attracts...
Particle production occurs near this point
log(a)
log(t)

If quantum effects in the gravitational field near a bounce produce enough matter, then the closed Universe can reach a size at which dark energy becomes dominant and expands to infinity.



Otherwise, the Universe contracts to another bounce (with larger scale factor) at which it produces more matter, and expands again.

Was Our Universe Born in a Black Hole?

Charles Peterson, Mechanical Engineering
Mentors: Dr. Nikodem Poplawski & Dr. Chris Haynes

BACKGROUND

Black holes (regions of space from where nothing can escape) form from massive stars that collapse because of their gravity.

The Universe is expanding, like the 3-dimensional analogue of the 2-dimensional surface of a growing balloon.

Problem. According to general theory of relativity, the matter in a black hole collapses to a point of infinite density (singularity). The Universe also started from a point (Big Bang). But infinities are unphysical.

Solution: Einstein-Cartan theory. Adding quantum-mechanical angular momentum (spin) of elementary particles generates a repulsive force (torsion) at extremely high densities which opposes gravitational attraction and prevents singularities.

HYPOTHESIS

We argue that the matter in a black hole collapses to an extremely high but finite density, bounces, and expands into a new space (it cannot go back). Every black hole, because of torsion, becomes a wormhole (Einstein-Rosen bridge) to a new universe on the other side of its boundary (event horizon).

If this scenario is correct then we would expect that:

- Such a universe never contracts to a point.
- This universe may undergo multiple bounces between which it expands and contracts.

Our Universe may thus have been formed in a black hole existing in another universe. The last bounce would be the Big Bang (Big Bounce). We would then expect that:

- The scalar spectral index (n_s) obtained from mathematical analysis of our hypothesis is consistent with the observed value n_s = 0.965 ± 0.006 obtained the Cosmic Microwave Background (CMB) data.

METHOD

To evaluate our expectations:

1. We wrote a code in Fortran programming language to solve the equations which describe the dynamics of the closed universe in a black hole (NP, arXiv:1410.3881) and then graph the solutions. These equations give the size (scale factor) *a* and temperature *T* of the universe as functions of time *t* (see Fig. 1).

$$\frac{\dot{a}^2}{c^2}+1=\frac{1}{3}\kappa\epsilon a^2, \quad \epsilon=h_\star T^4-\alpha h_{nf}^2 T^6$$
$$\frac{\dot{a}}{a}+\frac{\dot{T}}{T}=\frac{cK}{3h_{n1}T^3}, \quad K=\beta(\kappa\epsilon)^2$$

2. From the obtained graphs we found the values of the scalar spectral index n_s and compared them with the observed CMB value (see Fig. 2).

RESULTS

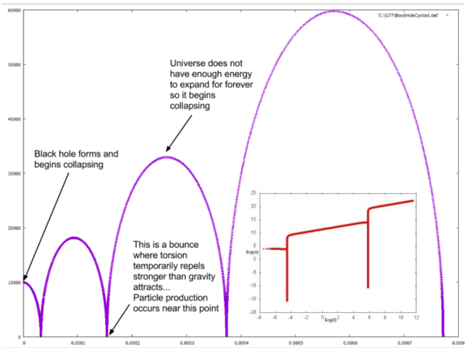


Fig. 1. Sample scale factor $a(t)$. Several bounces, at which a is minimum but always >0, may occur.

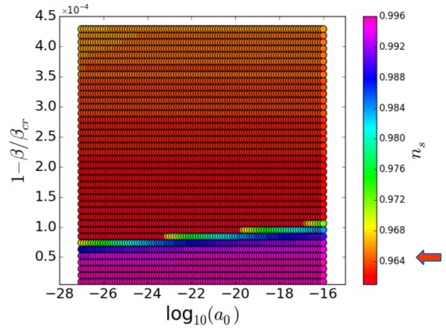


Fig. 2. The simulated values of n_s in our model are consistent with the observed CMB value n_s for a small range of β and a wide range of a_0 (m).

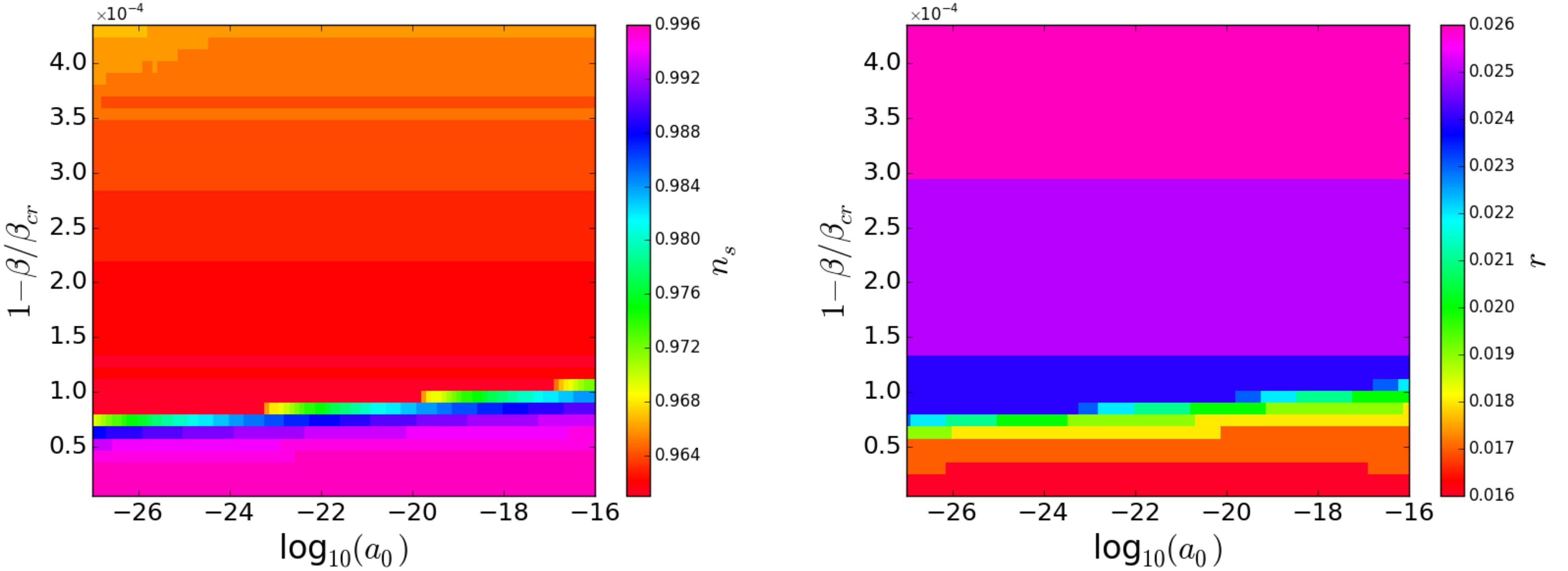
CONCLUSIONS

- The dynamics of the early universe formed in a black hole depends on the quantum-gravitational particle production rate β, but is not too sensitive to the initial scale factor a_0.
- Inflation (exponential expansion) can be caused by particle production with torsion if β is near some critical value β_{cr}.
- Our results for n_s are consistent with the 2015 CMB data, supporting our assertion that our Universe may have been formed in a black hole.

ACKNOWLEDGMENTS

I would like to thank my awesome teachers and mentors, Dr. Nikodem Poplawski and Dr. Chris Haynes, Dr. Shantanu Desai for his help, Carol Withers who organized the Summer Undergraduate Research Fellowship, and the donors who gave me the opportunity to pursue my research.

It is possible to find a scalar field potential which generates a given time dependence of the scale factor, and calculate the parameters which are measured in cosmic microwave background.



Consistent with Planck 2015 data.

S. Desai & NP, PLB **755**, 183 (2016)

Dark energy from torsion

Gen Relativ Gravit (2014) 46:1625

Affine theory of gravitation

Nikodem Popławski

Abstract We propose a theory of gravitation, in which the affine connection is the only dynamical variable describing the gravitational field. We construct a simple dynamical Lagrangian density that is entirely composed from the connection, via its curvature and torsion, and is a polynomial function of its derivatives. It is given by the contraction of the Ricci tensor with a tensor which is inverse to the symmetric, contracted square of the torsion tensor, $k_{\mu\nu} = S^{\rho}{}_{\lambda\mu} S^{\lambda}{}_{\rho\nu}$. We vary the total action for the gravitational field and matter with respect to the affine connection, assuming that the matter fields couple to the connection only through $k_{\mu\nu}$. We derive the resulting field equations and show that they are identical with the Einstein equations of general relativity with a nonzero cosmological constant if the tensor $k_{\mu\nu}$ is regarded as proportional to the metric tensor. The cosmological constant is simply a constant of proportionality between the two tensors, which together with c and G provides a natural system of units in gravitational physics. This theory therefore provides a physical construction of the metric as a polynomial function of the connection, and explains dark energy as an intrinsic property of spacetime.

Acknowledgments

UNH University Research Scholar Program

UNH Summer Undergraduate Research Fellowship

Prof. Ramesh Sharma

Dr. Shantanu Desai

Summary

Thank you!

- ❑ The ECSK gravity extends GR to be consistent with the Dirac equation which allows the orbital-spin angular momentum exchange. Spacetime must have both curvature and **torsion**.
- ❑ For fermionic matter at very high densities, torsion manifests itself as gravitational repulsion that prevents the formation of singularities in black holes and at the big bang. The big bang is replaced by a big bounce.
- ❑ Big-bounce cosmology with spin-torsion coupling and quantum particle production explains how inflation begins and ends, without hypothetical fields and with only one unknown parameter.
- ❑ Torsion can be the origin of the matter-antimatter asymmetry in the Universe and the cosmological constant.
- ❑ Torsion requires fermions to be extended, which may provide a UV cutoff for fermion propagators in QFT. Future work.